Applied Soft Computing Journal 73 (2018) 1–13

Contents lists available at ScienceDirect

Applied Soft Computing Journal

journal homepage: www.elsevier.com/locate/asoc

Meta fuzzy functions: Application of recurrent type-1 fuzzy functions

Nihat Tak *

Kirklareli University, Turkey
Ortabayir Mahallesi Caner Sokak No: 14/2 Kagithane, Istanbul, Turkey

HIGHLIGHTS

- MFFs are the first approach that aims to aggregate methods in functions by using FCM.
- The assumption of the MFFs is that a method has some information for a given dataset.
- The only need for applying the proposed method is to understand the FCM algorithm.
- MFFs gives more accurate results by aggregating the related methods in functions.

ARTICLE INFO

Article history:
Received 25 January 2018
Received in revised form 30 July 2018
Accepted 7 August 2018
Available online xxxx

MSC:
00-01
99-00

Keywords:
Fuzzy C-means clustering
Meta-analysis
Type-1 fuzzy functions
Hybridizing
Forecasting

ABSTRACT

The main objective of meta-analysis is to aggregate the results of multiple scientific studies on a specific topic. Instead of aggregating the results of different studies, different methods are aggregated with the help of fuzzy c-means clustering algorithm in the proposed method. Meta fuzzy functions are introduced in the paper. The idea of meta fuzzy functions is to aggregate the methods which are proposed for the same purpose; forecasting, prediction, etc. The study aggregates the models for the same method under different parameter specifications rather than aggregating different methods. Recently, recurrent type-1 fuzzy functions are introduced as an alternative forecasting method. The main advantages of recurrent type-1 fuzzy functions are that they are free of assumptions and rules. There are three parameters to be adjusted for recurrent type-1 fuzzy functions; the number of lags for AR(p), the number of lags for MA(q), and the number of clusters. The models for recurrent type-1 fuzzy functions with different parameter specifications are aggregated in the paper. The results show that it is possible to increase the forecasting performances of recurrent type-1 fuzzy functions in terms of both RMSE and MAPE.

1. Introduction

In this study, Meta Fuzzy Functions (MFFs) are introduced. One of the motivations of the study is the concept of meta-analysis. Meta-analysis was introduced by Glass [1] in 1976. He statistically aggregated the findings of 375 different psychotherapy outcome studies in his paper. Also, DerSimonian and Laird [2] defined that meta-analysis is a collection of analytic results for integrating the findings. The studies based on meta-analysis have become more popular in the last few decades. In the study, rather than aggregating the results of different studies for a purpose, the different methods for a purpose are aggregated. It has been shown that aggregation or hybridizing different methods have better forecasting or prediction accuracy. Thus, the main contribution of the paper is to aggregate as many methods as we can collect in functions. Aggregation of the methods is proceeded in the same sense of meta-analysis by looking at the outcomes/results of the methods. Aggregation of the methods are performed by using Fuzzy C-Means (FCM) clustering technique in the study. This is a first study in the literature that aims to aggregate methods in the notion of meta-analysis by using FCM.

The proposed method is formed with a set of outcomes of the methods for a specific topic, such as forecasting performances of methods for a time series dataset. The outcomes of the methods are clustered using Fuzzy C-Means (FCM) clustering algorithm which was introduced by Bezdek [3]. Using the degrees of memberships values of the methods for each cluster, MFFs are obtained. Finally, the function that has the best set of outcomes is selected as the best MFF. As an application, a forecasting method, Recurrent Type-1 Fuzzy Functions (R-T1FF), which was introduced by Tak et al. [4] is used.

T1FFs were introduced for classification and regression problems at first by Turksen [5] in 2008. First, Beyhan and Alici [6] and later, Aladag et al. [7] adapted T1FFs to time series forecasting problems. Beyhan and Alici [6] used an auto-regressive with exogenous input (ARX) model structure that was not able to search for the best model. Therefore, Aladag et al. [7] proposed a forecasting method

* Correspondence to: Kirklareli University, Turkey.
E-mail address: nihattak@gmail.com.

https://doi.org/10.1016/j.asoc.2018.08.009
1568-4946/

to search for the best model by adapting Autoregressive (AR) model into their algorithm. Tak et al. [4] in 2017 introduced another method with T1FFs including also Moving Average (MA) model with recurrent learning structure into their algorithm. Eventually, R-T1FFs outperformed other two methods, which use T1FFs, in terms of RMSE values.

The purpose of the study is to aggregate the methods for the same purpose into functions. The idea behind aggregating the methods is the assumption that each method has much, partial or no information for a given dataset. Thus, while the methods that perform better will be collected into one function, the methods that perform worse will be collected into another function. Our aim is to obtain better outcomes using the power of many methods or, at least, obtain the best method among many.

The remainder of this article is organized as follows. In the second section, two algorithms are given; MFFs and MFFs of T1FFs. In order to evaluate the performance of the proposed method, some applications are given in the third section. Finally, the conclusions and remarks are given in Section 4.

2. Proposed method

The questions like "Which method should we choose for a dataset?" and "Please explain why the proposed method does not outperform other methods?" lead us to come up with the proposed method, MFFs. The main scope of the proposed method is to aggregate as many methods as we can collect into functions and, eventually, to get better outcomes. Thus, the input matrix of the MFFs consists of the outcomes of the methods which were previously introduced. Using the outcomes of the methods as their characteristics, the methods are clustered by using FCM. Finally, the best cluster (function) is selected as the MFF_{best}. Although there are different fuzzy clustering techniques such as FCM [3], Possibilistic C-Means (PCM) [8], Fuzzy Possibilistic C-Means (FPCM) [9], Possibilistic Fuzzy C-Means (PFCM) [9], and Interval Type-2 Fuzzy Possibilistic C-Means (IT2FPCM) [10], FCM is preferred because of its fame and simplicity. In other words, FCM is just one way to obtain the weights of the methods in functions. Using another fuzzy clustering technique in the setup of the MFFs might be the future work.

Two algorithms are introduced in this section. The first one is the main algorithm that the paper aims to propose and the second algorithm is an application of MFFs.

2.1. Algorithm 1: meta fuzzy functions

There are three components in the main algorithm that needs to have further clarification. The first one is the clarification of "function". A function consists of the combination of the methods. When we say "a function" we mean "a cluster" in FCM. The second one is the weights of the methods in functions. Weights of the methods are simply obtained from the degrees of memberships of a method in a cluster. The third one is the best meta fuzzy function (MFF_{best}). Because we have as many functions as the number of clusters, we are looking for a function that has the best evaluation criteria. In this case, the function that has the best evaluation criteria is called MFF_{best} and future estimations or forecasts are calculated with (MFF_{best}).

- Step 1. Determination the existing related methods for the problem (i.e forecasting) and training of the related methods.
  - Step 1.1. Determine m existing methods for the problem.
  - Step 1.2. The dataset is divided into two sets; training and test datasets.

$$X = \left[X_{ij}\right], i = 1, 2, \ldots, p; j = 1, 2.., n \quad (1)$$

$$X_{train} = \left[X_{ij}\right], i = 1, 2, \ldots, p; j = 1, 2.., ntrain \quad (2)$$

$$X_{test} = \left[X_{ij}\right], i = 1, 2, \ldots, p; j = ntrain+1, ntrain+2, \ldots, n \quad (3)$$

  - Step 1.3. Train the related existing methods for a given dataset by using X_{train}.
  - Step 1.4. Obtain the outcomes (i.e forecasts) by using the trained methods for the test dataset, X_{test}. The input matrix of the MFFs are the collection of outcomes of the trained methods.

$$Z = \left[Z_{ij}\right], i = 1, 2, \ldots, ntest; j = 1, 2..., m \quad (4)$$

where Z_{ij} is the outcomes of the ith data point for jth method.

$$Z = \begin{bmatrix} Z_{1,1} & Z_{1,2} & . & . & Z_{1,ntest} \\ Z_{2,1} & Z_{2,2} & . & . & Z_{2,ntest} \\ . & . & . & . & . \\ . & . & . & . & . \\ Z_{m,1} & Z_{m,2} & . & . & Z_{m,ntest} \end{bmatrix}$$

- Step 2. The input matrix Z is divided into two, training (Z_{train}) and test (Z_{test}) sets. The training set is used to determine the weights of the methods in functions and the test set is used to evaluate the performance of MFFs.

$$Z_{train} = \left[Z_{ij}\right], i = 1, 2, \ldots, ntrain1; j = 1, 2..., m \quad (5)$$

$$Z_{ntest} = \left[Z_{ij}\right], i = ntrain1, ntrain1+1, \ldots, ntest; j = 1, 2..., m \quad (6)$$

- Step 3. Determining the weights of the methods in functions. The input matrix Z_{train} is clustered by using FCM. The degrees of memberships in each cluster are used to determine the weights of the methods in functions. In this case, a cluster represents a function.
  - Step 3.1. Determine the fuzziness index parameter, number of clusters and initial cluster centers.
  - Step 3.2. Calculate the membership value with the formula

$$\mu_{ik} = \left[\sum_{j=1}^{c}\left(\frac{d(z_k, v_i)}{d(z_k, v_j)}\right)^{\frac{2}{m-1}}\right]^{-1}, \quad i = 1, 2, \ldots, c; k = 1, 2, \ldots, n \quad (7)$$

under the constraint; $\sum_{i=1}^{c}\mu_{ik} = 1$, $if\, \mu_{ik} < \alpha - cut$, then μ_{ik} value will be taken as zero. Z is the input matrix, v is the cluster centers, $d(.)$ stands for Euclidean distance function, c is the number of clusters, and m is the fuzziness parameter in Eqs. (7)–(8).

  - Step 3.3. Calculate the new cluster centers.

$$v_i = \frac{\sum_{k=1}^{n}\mu_{ik}^{m}z_k}{\sum_{k=1}^{n}\mu_{ik}^{m}} \quad (8)$$

  - Step 3.4. Repeat Step 2 and Step 3 until the difference of clusters between two iterations drops under some threshold or the number of iterations is reached.

- Step 4. Obtaining the meta fuzzy functions.
  MFFs, which are given in Eq. (9), are obtained by using the degrees of memberships that are calculated for the training set.

$$MFF_i(z) = \sum_{j=1}^{m} w_{ij} z_j, i = 1, 2, \ldots, c \quad (9)$$

$$w_{ij} = \frac{\mu_{ij}}{\sum_{j=1}^{m} \mu_{ij}}, i = 1, 2, \ldots, c \quad (10)$$

  where MFF_i stands for the ith meta fuzzy function, μ_{ij} stands for the degree of membership value of the jth method in ith cluster and c represents the number of clusters.
- Step 5. Repeat Step 3–4 for different m and c.
- Step 6. Selecting the best combination of the methods in a function.
  The function that has the best evaluation criteria is chosen as the best meta fuzzy function (MFF_{best}).
- Step 7. The outcomes of the MFFs' system is calculated by using the MFF_{best} for the test set, Z_{test}.

$$F = MFF_{best}(z_{test}) \quad (11)$$

  where F is the outcomes of the MFFs.

Algorithm 1: Pseudo code of the MFFs

```
Determine the methods for the purpose
Specify the length of the training and test datasets
Train the related methods by using the training dataset (Y_train)
Obtain the outcomes from the trained methods for the test
dataset (Y_test)
Use the outcomes of the methods as the input matrix (Z) for MFFs
Divide Z into two as Z_train and Z_test
Initialize the maximum number of clusters(c) and fuzzy index
parameter (m)
while (α <m ) do
    while (i < max number of functions/clusters (c)) do
        Use FCM for Z_train to determine the weights of each method
        in each function
        Obtain the MFFs by using Equation 9
        Calculate the evaluation criteria of MFFs
        j = j + 1
    α = α + 0.1
Return the best function ( MFF_best ) that has the best evaluation
criteria
Calculate the outcomes (F) by using MFF_best
```

2.2. Algorithm 2: MFFs of R-T1FFs

Because fuzzy set theory deals with the uncertainty, there have been many studies employing fuzzy set theory to forecasting problems. Some of these studies are conducted by Song and Chissom [11,12], Chen [13] Chen and Zhang [14], Gupta et al. [15], and Aladag et al. [16]. In the literature, it has been also shown that the combination of methods improves the forecasting performances. Makridakis [17] has pointed out that combining reduces the variance of forecasting errors. Thus, the empirical finding of combining improves forecasting accuracy holds true. Bates and Granger [18] combined two forecasting methods in their study and got better forecasting performances than both forecasting methods. Granger and Ramanathan [19] proposed 3 approaches for combining three forecasting methods. Besides, some of the studies on combining forecasts are conducted by Bunn [20], Newbold and Granger [21], Zou and Yang [22], and Aladag et al. [23]. In this application, the models of a recently introduced time series forecasting method (R-T1FFs) are aggregated with MFFs. Aggregating the different forecasting methods is left for future work.

The important issue with recurrent systems is stability. R-T1FFs are very sensitive the initial starting points and different parameter specifications as well. Therefore, the model that has the best evaluation criteria is searched iteratively with different starting points and parameter specifications to ensure stability in R-T1FFs. In, R-T1FFs, there are 3 parameters, which has an effect on the forecasting performances, to be adjusted; the number of clusters, the number of lags for AR(p) model, and the number of lags for MA(q) model. The pseudo code of the R-T1FFs based on MFFs is given Algorithm 2.

Algorithm 2: Pseudo code of MFFS of R-T1FFs

```
Specify the length of the training and test datasets
Specify the maximum number of clusters and number of lags for
AR and MA for R-T1FFs
while i<c do
    while j<p do
        while k<q do
            Train the R-T1FFs by using training dataset
            if RMSE < cut-off then
                Save the forecasts in Z
            k=k+1
        j=j+1
    i=i+1
Use the outcomes of the m models as the input matrix (Z) for
MFFs
Divide Z into two as Z_train and Z_test
Initialize the maximum number of clusters and fuzzy index
parameter
while (α <m ) do
    while (i < max number of functions/clusters (c)) do
        Use FCM for Z_train to determine the weights of each
        forecasting model in each function/cluster
        Obtain the MFFs by using Equations 20
        Calculate the evaluation criteria of MFFs
        i = i + 1
    α = α + 0.1
Return the function (MFF_best) that has the best evaluation criteria
Calculate the forecasts (F) by using MFF_best for Z_test
```

- Step 1. The time series dataset is divided into two as training and test datasets by using the block partitioning technique.

$$Y = [Y_i], i = 1, 2, \ldots, t \quad (12)$$

$$Y_{train} = [Y_i], i = 1, 2.., ntrain \quad (13)$$

$$Y_{test} = [Y_i], i = ntrain + 1, ntrain + 2, \ldots, n \quad (14)$$

- Step 2. Obtaining the input matrix of MFFs from R-T1FFs.
  - Step 2.1. There are 3 parameters, which have effect on forecasting performances, to be adjusted for R-T1FFs, the number of clusters and the number of lags for AR and MA for R-T1FFs forecasting approach.
  - Step 2.2. Obtain m different forecasting models from R-T1FFs with different parameter specifications for Y_{train} by putting a restriction on RMSE value. R-T1FFs is a forecasting method that has been recently introduced. The detailed steps of R-T1FFs are found in [4].

- Step 2.3. Obtain the forecasts of m models The input matrix (Z) are constituted of the forecasts of the m models for MFFs.

$$Z = \left[Z_{ij}\right], i = 1, 2, \ldots, ntest; j = 1, 2, \ldots, m \quad (15)$$

Step 1–2 are proceeded to determine the input matrix of the MFFs. In this case, because we have a forecasting problem, the models of R-T1FFs with different parameter specifications are aggregated in functions. The rest of the algorithm is the main algorithm of MFFs but modified for the forecasting problem.

- Step 3. The input matrix Z is divided into two, training (Z_{train}) and test (Z_{test}) sets. The training set is used to determine the weights of the models of R1TFFs in functions and the test set is used to evaluate the performance of MFFs.

$$Z_{train} = \left[Z_{ij}\right], i = 1, 2, \ldots, ntrain1; j = 1, 2..., m \quad (16)$$

$$Z_{ntest} = \left[Z_{ij}\right], i = ntrain1, ntrain1 + 1, \ldots, ntest; j = 1, 2..., m \quad (17)$$

- Step 4. Initialize the maximum number of clusters(c) and fuzziness parameter (m). There are some studies in the literature on specifying the fuzziness parameter.

Many studies in literature have been conducted by researchers based on the selection of fuzziness parameter. One of them is introduced by Pal and Bezdek [24], which points out that the optimal value of m is limited to [1.5,2.5]. On the other hand, Ozkan and Turksen [25] identified that the upper and lower value of m is 1.4 and 2.6, respectively. Chan and Cheung [26] suggested that the value of m should be in [1.25, 1.75] in a study of word recognition. Also, Bezdek [27] proposed that the optimum selection of m is 2.

- Step 5. The weights of each model in each function is obtained by using the degree of memberships values in FCM.
  - Step 5.1. Determine the fuzziness index parameter, number of clusters and initial cluster centers.
  - Step 5.2. Calculate the membership value with the formula

$$\mu_{ik} = \left[\sum_{j=1}^{c}\left(\frac{d(z_k, v_i)}{d(z_k, v_j)}\right)^{\frac{2}{m-1}}\right]^{-1}, \quad i = 1, 2, \ldots, c; k = 1, 2, \ldots, n \quad (18)$$

under the constraint; $\sum_{i=1}^{c}\mu_{ik} = 1$, if $\mu_{ik} < \alpha - cut$, then μ_{ik} value will be taken as zero. Z is the input matrix, v is the cluster centers, $d(.)$ stands for Euclidean distance function, c is the number of clusters, and m is the fuzziness parameter in Eqs. (1)–(2).

  - Step 5.3. Calculate the new cluster centers.

$$v_i = \frac{\sum_{k=1}^{n}\mu_{ik}^{m}z_k}{\sum_{k=1}^{n}\mu_{ik}^{m}} \quad (19)$$

  - Step 5.4. Repeat Step 2 and Step 3 until the difference of clusters between two iterations drops under some threshold or the number of iterations is reached.

- Step 6. Using the degree of memberships values and the forecasts that are obtained from different models, MFFs are calculated with the formula given in Eq. (6).

$$MFF_i(z) = \sum_{j=1}^{m} w_{ij}z_j, i = 1, 2, \ldots, c \quad (20)$$

$$w_{ij} = \frac{\mu_{ij}}{\sum_{j=1}^{m}\mu_{ij}}, i = 1, 2, \ldots, c \quad (21)$$

where MFF stands for the meta fuzzy function and c stands for the number of clusters.

- Step 7. Step 6 is repeated for the number of clusters times.
- Step 8. Step 5–7 is repeated for different m and c specifications.
- Step 9. The function that has the minimum Root Mean Squared Errors (RMSE) or Mean Absolute Percentage Errors (MAPE) is selected as the MFF_{best}.
- Step 10. The forecast are calculated by using MFF_{best}.

$$F = MFF_{best}(z_{test}) \quad (22)$$

where F is the forecasts of the MFFs.

3. Evaluation

9 real world time series datasets were analyzed to evaluate the performance of the MFFs of R-T1FFs. All calculations are done using R, a statistical programming language. The first dataset is the Australian Beer Consumption (ABC) dataset [28] which were observed for each quarter from 1956 to 1994. The next 4 datasets are from the Istanbul Stock Exchange (BIST100) [29]. The observations of four datasets were observed daily for the first half of a year from 2009 to 2012. The last 4 datasets are from Dow Jones stock exchange. The observations of the Dow Jones index are daily measured year by year from 2010 to 2013. The summary of the datasets and parameter selection criteria of R-T1FFs are given Table 1. The methods are evaluated using MAPE and RMSE. RMSE and MAPE values are two commonly used evaluation metric in the literature for forecasting purposes. RMSE aims to measure the magnitude of the error. It uses the actual and predicted values of a time series in a distance function to measure. MAPE, as well, uses predicted and actual values of a time series to measure the average magnitude of the errors. Both error measures, aim to obtain the average model forecasting error in observations of a time series and both criteria range between 0 and infinity. The lower RMSE/MAPE value means the better forecasting accuracy for both metric. The formulations of RMSE and MAPE values are given in Eqs. (23)–(24)

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(x_t - \widehat{x}_t)^2} \quad (23)$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{x_t - \widehat{x_t}}{x_t}\right| \quad (24)$$

where x_t is the actual values of the time series and $\widehat{x_t}$ is the forecasts.

The stock index data are known as a complex time series because there are many factors that can affect stock prices. Because of mostly non-linear structure of stock index datasets, statistical approaches usually fail to give accurate results. Therefore, non-statistical approaches commonly use stock index datasets for the evaluation of their proposed methods. IEX and DowJones datasets are chosen in this sense and to be able to compare the forecasting results of MFFs of R-T1FFs with the existing methods that used the same datasets previously. ABC is another commonly used dataset by researchers and it contains both seasonality and trend. Therefore, ABC, IEX, and DowJones datasets are chosen to be able to compare the forecasting accuracy of the MFFs of R-T1FFs with the existing ones.

In order to perform MFFs, R-T1FFs is chosen as an application method. In other words, different models that are obtained from

Table 1
Summary of the datasets and parameter selection criteria of R-T1FFs.

	Series/Year	Obs.	AR	MA	Clusters numbers	ntest
1	ABC	147	1–10	1–2	2–10	16
2	BIST100/2009	103	1–5	1–2	2–5	15
3	BIST100/2010	104	1–5	1–2	2–5	15
4	BIST100/2011	106	1–5	1–2	2–5	15
5	BIST100/2012	106	1–5	1–2	2–5	15
6	DowJones/2010	252	1–5	1–2	2–5	10
7	DowJones/2011	251	1–5	1–2	2–5	10
8	DowJones/2012	250	1–5	1–2	2–5	10
9	DowJones/2013	252	1–5	1–2	2–5	10

R-T1FFs will be aggregated with MFFs. R-T1FFs is a recently introduced forecasting method by Tak et al. [4]. There are 3 parameters to be adjusted for R-T1FFs, the number of lags for AR(p) and MA(q) and the number of clusters, c. Although there are numerous studies on determining the optimum number of clusters for FCM, there is no consensus on which one the best is. Therefore, rather than using a method for determining the optimum number of clusters, the one that gives the minimum SSEs is chosen as the optimum number of clusters for R-T1FFs.

There are many information criteria for selecting the optimum lag length for ARMA model; root mean squares (RMSE), Schwartz information criteria (SIC), Hannan–Quinn criterion (HQ), Akaike information criteria (AIC), and etc. There is no consensus on the question of "which criteria the best is to determine the optimum number of lags". Thus, one approach to this is to use all criteria cited above and chose the one that has the smallest lag length. However, in R-T1FFs approach, sum of squares errors is used to determine the number of lags for AR and MA models. The optimum numbers of lags for AR and MA are searched iteratively and the ones that have the smallest SSEs are chosen. However, in the evaluation part of the proposed method, we are not looking for the best sets of lags for AR and MA models, we are randomly selecting models with different parameter specifications because we are aggregating numerous models that are obtained with different sets of lags lengths and the cluster numbers. Optimum numbers of p, q, and c, which gives the minimum SSEs, are searched iteratively in R-T1FFs. For each dataset, some cut-off (RMSE/MAPE) value is determined and 10 models for each dataset under some RMSE/MAPE value is stored. The one that has the best RMSE/MAPE, the one that has the worst RMSE/MAPE value, and randomly 8 models are selected from the storage as an input matrix (Z) for MFFs.

There are two parameters to be adjusted in MFFs; the fuzziness parameter (m) and the number of clusters (c). The fuzziness parameter is varied from 1.3 and 3 with an increase rate of 0.1 and the number of clusters is varied from 2 to 5. The m and c which give the best outcomes, in terms of RMSE/MAPE, are selected for MFFs.

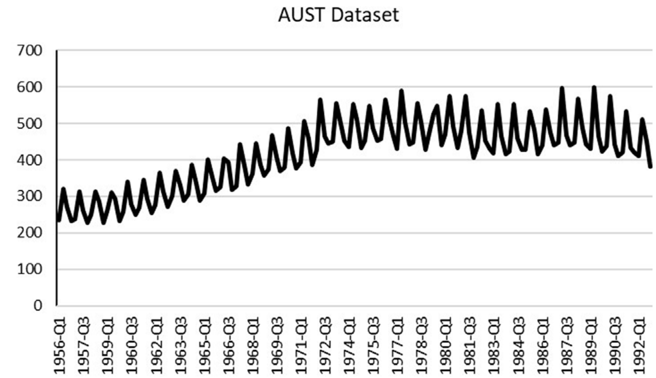

Fig. 1. Line plot of ABC dataset.

3.1. ABC dataset

In the first evaluation, the ABC dataset was used. This dataset consists of 148 observations that were observed quarterly from 1956 to 1994. The line plot of the ABC dataset is given in Fig. 1.

In the training phase of the R-T1FFs, the last 16 observations were left for the test set (Y_{test}) for ABC and 10 models were obtained with different parameter (p, q, c) specifications. 16 data points are forecasted by using trained models and given in Table 2. In order to obtain MFFs and MFF_{best}, the first half of 16 forecasts of the models are used as training set, Z_{train} (see Table 3). The last 8 observations (Z_{test}) (see Table 4) are used to be able to compare the performance of the proposed method with the existing methods. Winter's multiplicative exponential smoothing (WMES), SARIMA [30], Feed-Forward Artificial Neural Network (FFANN), Adaptive-Network-based Fuzzy Inference Systems (ANFIS) [31], Modified ANFIS (MANFIS) [32], Autoregressive ANFIS (AR-ANFIS) [33], and R-T1FF [4], which used the ABC datasets as applications previously, are selected to compare the performance of MFFs and these results are quoted from [4].

The algorithm searched for the best models when the number of clusters is varied from 2 to 5 and the fuzziness parameter is varied from 1.3 to 3 with the increase rate of 0.1 for ABC dataset. Under these conditions, the function which has the minimum RMSE value is obtained when the number of clusters (functions) is 3 and the fuzziness parameter is 1.4. The weights of the models, which are

Table 2
10 different forecasting results of the R-T1FF for the ABC dataset.

Actual	a	b	c	d	e	f	g	h	i	j
430.5	441.851	470.486	471.499	468.161	467.927	462.902	437.102	442.820	410.151	480.045
600	524.271	543.559	568.204	539.864	528.573	554.725	554.732	554.440	594.661	566.570
464.5	403.488	422.800	517.472	479.828	493.606	493.919	461.061	477.330	492.278	483.811
423.6	481.151	491.726	404.833	430.096	419.131	441.908	448.495	443.470	379.198	417.403
437	427.204	455.920	422.259	500.954	499.691	441.356	419.611	422.460	435.409	460.859
574	543.018	563.500	605.301	530.089	524.607	585.824	579.569	571.210	600.818	586.967
443	386.201	399.871	467.588	471.466	458.572	456.075	443.136	463.850	450.811	430.541
410	463.752	478.193	355.220	419.191	418.268	409.686	418.853	410.470	372.250	430.060
420	427.073	454.269	431.630	456.973	458.331	451.447	428.554	420.020	416.309	443.438
532	518.738	537.956	567.749	503.910	517.483	553.002	555.769	551.340	570.405	568.157
432	370.634	384.878	418.941	455.676	424.240	423.280	424.715	436.370	451.239	404.538
420	439.149	454.504	343.182	393.059	406.850	394.730	400.075	390.610	361.471	428.945
411	407.519	433.294	427.059	428.594	444.784	443.988	409.558	398.690	416.145	419.524
512	493.369	511.786	526.537	498.236	497.189	522.275	511.579	517.620	506.062	528.857
449	369.602	384.930	433.857	457.612	436.258	424.613	421.309	430.760	450.367	402.704
382	435.946	451.793	392.393	377.730	405.179	417.800	415.594	408.270	374.640	437.313
RMSE	47.000	44.000	34.400	31.870	33.560	24.805	19.561	19.210	25.976	29.079

Table 3
Training dataset (Z_{train}) for MFFs.

Actual	a	b	c	d	e	f	g	h	i	j
430.5	441.851	470.486	471.499	468.161	467.927	462.902	437.102	442.820	410.151	480.045
600	524.271	543.559	568.204	539.864	528.573	554.725	554.732	554.440	594.661	566.570
464.5	403.488	422.800	517.472	479.828	493.606	493.919	461.061	477.330	492.278	483.811
423.6	481.151	491.726	404.833	430.096	419.131	441.908	448.495	443.470	379.198	417.403
437	427.204	455.920	422.259	500.954	499.691	441.356	419.611	422.460	435.409	460.859
574	543.018	563.500	605.301	530.089	524.607	585.824	579.569	571.210	600.818	586.967
443	386.201	399.871	467.588	471.466	458.572	456.075	443.136	463.850	450.811	430.541
410	463.752	478.193	355.220	419.191	418.268	409.686	418.853	410.470	372.250	430.060

Table 4
Training dataset (Z_{test}) for MFFs.

Actual	a	b	c	d	e	f	g	h	i	j
420	427.073	454.269	431.630	456.973	458.331	451.447	428.554	420.020	416.309	443.438
532	518.738	537.956	567.749	503.910	517.483	553.002	555.769	551.340	570.405	568.157
432	370.634	384.878	418.941	455.676	424.240	423.280	424.715	436.370	451.239	404.538
420	439.149	454.504	343.182	393.059	406.850	394.730	400.075	390.610	361.471	428.945
411	407.519	433.294	427.059	428.594	444.784	443.988	409.558	398.690	416.145	419.524
512	493.369	511.786	526.537	498.236	497.189	522.275	511.579	517.620	506.062	528.857
449	369.602	384.930	433.857	457.612	436.258	424.613	421.309	430.760	450.367	402.704
382	435.946	451.793	392.393	377.730	405.179	417.800	415.594	408.270	374.640	437.313
RMSE	41.731	41.973	32.193	22.455	22.286	25.531	19.319	17.546	25.986	32.114
MAPE	0.074	0.083	0.0546	0.044	0.046	0.055	0.0354	0.0333	0.0389	0.0638

Table 5
The weights of the models in functions and RMSE values of the functions for Z_{train}.

Model	MFF_1	MFF_2	MFF_3
a	0.496	0.000	0.000
b	0.495	0.000	0.000
c	0.000	0.476	0.000
d	0.000	0.000	0.174
e	0.000	0.000	0.171
f	0.000	0.000	0.176
g	0.000	0.000	0.145
h	0.000	0.000	0.164
i	0.001	0.495	0.002
j	0.008	0.029	0.166
RMSE	47.452	27.484	24.023*
MAPE	0.091	0.053	0.041*

Table 6
Forecasts of MFFs for ABC dataset and RMSE and MAPE values of the functions.

Test	MFF_1	MFF_2	MFF_3
9	440.67	424.39	443.69
10	528.69	569.08	541.04
11	378.01	434.51	428.40
12	446.62	354.75	402.23
13	420.39	421.43	424.86
14	502.78	516.47	512.52
15	377.51	441.12	429.30
16	443.77	384.92	409.91
RMSE	40.56	27.06	17.07*
MAPE	0.0757	0.0376	0.0344*

calculated with Eq. (21), in functions and the RMSE and MAPE values of the functions for Z_{train} are given in Table 5.

Table 5 reveals that the best MFF is the third function in terms of RMSE and MAPE values. Therefore, forecasts are calculated by using $MFF_{best} = MFF_3$. MFF_{best} is given in Eq. (25) and the forecasting results of MFF_{best} and the other functions are given in Table 6.

$$MFF_{best} = MFF_3 = 0.174 * d + \cdots + 0.102 * i + 0.166 * j \tag{25}$$

In order to evaluate the performance of the proposed method with the existing methods Table 7 is given. In terms of RMSE and MAPE values, it is obvious that the best forecasting results are obtained from the proposed method.

3.2. BIST100 datasets

There are 4 datasets year by year from 2009 to 2012 for BIST100. To obtain 10 models from R-T1FF, the number of clusters was varied from 2 to 5, the AR lag was varied from 1 to 5, and the MA lag was varied from 1 to 2. The last 15 observations for all BIST100 datasets are selected as the test dataset (Y_{test}). The input matrix (Z) of MFFs are obtained from the 15 point forecasts of 10 models. The first 8 observations of Z is selected as the training matrix (Z_{train}) in order to determine the weights of the models in functions and MFF_{best}. The last 7 observations are left for test dataset (Z_{test}) to evaluate the performance of the proposed method with the existing methods. ARIMA [30], Exponential Smoothing (ES) [34], Multilayer Perceptron ANN (MLP-ANN), Fuzzy Functions (FF) [5], Fuzzy Time Series Network (FTS-N) [35], and R-T1FF [4], which used the BIST100 datasets as applications previously, are selected to compare the performance of MFFs.

For MFFs, there are two parameters to be adjusted; fuzziness value (m) and the number of functions (c). The number of functions is varied from 2 to 5 and the fuzziness value is varied from 1.3 to 3 with the increase rate of 0.1. The detailed results are given for the datasets of 2009 for BIST100. Only the input matrices and formal comparative tables are given for the rest of the datasets of BIST100.

3.2.1. 2009

For BIST100 dataset in 2009, 15 forecasts of 10 models are used as the input matrix, Z. The first 8 observations of Z is used as training matrix (Z_{train}) and the last 7 observations are used as the test dataset (Z_{test}) and they are given in Tables 8 and 9, respectively.

The best MFFs are obtained when $m = 2.4$ and $c = 3$. The weight matrix that are calculated by FCM for Z_{train} is given in Table 3.

The best function is determined from MAPE values of the MFFs for Z_{train}. In this case, MFF_{best} is selected as MFF_2 (see Table 10). Thus, the forecasts of the proposed methods is obtained by using MFF_{best}. MFF_{best} is given in Eq. (26) and the corresponding forecasting results and the other functions are given in Table 11.

$$MFF_{best} = MFF_2 = 0.04 * a + \cdots + 0.20 * i + 0.21 * j \tag{26}$$

6 existing forecasting methods are compared with the proposed method in Table 12. It is obvious that the MFF_{best} of the proposed

Table 7
Forecasting results and RMSE/MAPE values of existing methods and MFF_{best}.

Test	WMES	SARIMA	FFANN	ANFIS	MANFIS	AR-ANFIS	T1FF	MFF_{best}
420	465.4	461.01	448.87	445.0127	430.31	419.428	431.60	443.69
532	589.74	588.96	560.04	562.94	565.18	570.378	559.41	541.04
432	514.96	496.77	447.01	459.14	452.05	440.589	444.08	428.40
420	455.89	454.64	408.64	416.16	392.14	400.06	394.99	402.23
411	471.15	465.46	428.11	431.70	419.33	413.925	409.72	424.86
512	597	594.71	537.70	544.98	536.88	549.318	525.60	512.52
449	521.28	501.67	438.43	444.31	446.32	441.676	438.91	429.31
382	461.46	459.17	420.58	426.01	406.64	413.59	409.07	409.91
RMSE	67.032	60.06	23.79	26.95	21.47	23.44	18.30	17.07*
MAPE	0.147	0.13	0.050	0.054	0.0424	0.040	0.036	0.034*

Table 8
Training dataset (Z_{train}) for 2009.

Test	a	b	c	d	e	f	g	h	i	j
32806	32664.29	33398.2	32898	33420.2	33072.6	33200.3	33170.4	33132.9	33731.4	33269.3
32203	32594.61	32876.5	32822.2	33109.5	32938.4	32647.9	32708	32933.5	32558.4	32121.6
33043	31662.56	31685.1	32171.8	32768.5	32601.9	32883.6	32234.4	32442.1	32736.7	32699.8
32829	32786.4	32774.8	33050.5	32815.8	33119.5	32544.1	33142.7	33207.1	32678	32648.3
33095	33133.76	32884	32897.1	33230.3	32837.1	33271	33245.2	33012.1	33769.5	33187.7
33485	33064.32	33037.2	33188.1	33250.8	33252.2	33230.5	33446.1	33274.8	33622.9	33411.3
33666	33160.11	33561.6	33578.9	33641.9	33604.7	33424.3	33852	33661.3	33970.6	33793.7
35140	33657.81	33644	33752.8	33845	33743.4	33833	34062.1	33848.8	34099.2	33867

Table 9
Test dataset (Z_{test}) for 2009.

Test	a	b	c	d	e	f	g	h	i	j
34721	35364.49	35047.95	34687.79	34455.7	35052.9	34458.45	35196.19	34352.33	35180.2	34534.4
35015	35261.99	34516.6	34836.9	34992.4	34797.1	35348.2	34882.9	34902	35495.7	34964.2
35408	35335.15	35346	35112	35140	35117.1	35408.5	35411.1	35201.7	35026.4	35076.3
34861	34858.44	35268.2	35522.7	35326.4	35488.4	35211.6	35506.8	35593.3	34819.7	34849.3
35169	35317.37	34982	34991.2	34751	34923.5	35468.3	34821.5	35042.8	34799.2	34739.5
35021	35280.65	35108.8	35292.7	34896.7	35244.3	35387.4	35169.4	35354.5	34849.2	34420.1
35003	34997.85	35075.1	35148.9	34808.7	35106.8	35228.2	34968.7	35203.9	35200.3	34648
RMSE	395.8234	420.1002	303.8493	270.3831	343.01	394.37	373.504	392.70	345.84	200,9087
MAPE	0.0095	0.0108	0.00683	0.00739	0.0087	0.0097	0.00935	0.0093	0.00861	0.00487

Table 10
Weights of the models in functions and RMSE values of the functions for Z_{train}.

Model	MFF_1	MFF_2	MFF_3
a	0.04	0.04	0.28
b	0.06	0.06	0.22
c	0.12	0.05	0.14
d	0.11	0.13	0.06
e	0.18	0.04	0.05
f	0.07	0.18	0.06
g	0.12	0.09	0.08
h	0.21	0.00	0 .00
i	0.05	0.20	0.06
j	0.05	0.21	0.06
RMSE	575.64	531.48*	637.11
MAPE	0.012	0.011*	0.013

methods outperforms other methods in terms of both RMSE and MAPE values.

Table 11
Forecasting results and RMSE/MAPE values of the functions.

Model	MFF_1	MFF_2	MFF_3
9	35084.72	34740.53	34865.10
10	34937.89	35112.87	34974.52
11	35205.86	35195.13	35262.91
12	35373.75	35112.96	35169.56
13	34964.67	34949.34	35051.79
14	35166.00	34961.31	35142.70
15	35067.51	34996.57	35032.74
RMSE	269.64	156.12*	154.96
MAPE	0.0064	0.0035*	0.0037

3.2.2. 2010

The 15 forecasts of the 10 models are used as the input matrix (Z) of MFFs. The input matrix is divided into two training (Z_{train}) and test (Z_{test}) sets. Tables 13–14 represents the training and test sets respectively for 2010.

Table 12
Forecasting results and RMSE/MAPE values of existing methods and MFF_{best}.

Test	ARIMA	ES	MLP-ANN	FF	FTS-N	R-T1FF	MFF_{best}
34721	35140	35139.7	34926.15	35353.47	34559.35	34676.64	34740.53
35015	34721	34720.55	34547.22	35065.66	34999.72	35140.3	35112.87
35408	35015	35014.5	34887.24	35247.61	35042.18	35396.82	35195.13
34861	35408	35407.68	35108.24	35720.93	35257.93	35186.02	35112.96
35169	34861	34860.56	34727.41	35196.88	35186.01	34850.95	34949.34
35021	35169	35168.73	35002.28	35444.51	35103.95	34719.4	34961.31
35003	35021	35021.01	34844.72	35369.76	35103.37	34799.38	34996.57
RMSE	344.9114	344.9628	340.929	460.1925	218.7582	225.8351	156.12*
MAPE	0.0087	0.0087	0.0084	0.0103	0.0046	0.0054	0.0035*

Table 13
Training dataset (Z_{train}) for 2010.

Test	a	b	c	d	e	f	g	h	i	j
56448	55203.7	54002.99	54505.2	54651.9	55712.9	54246	55156.3	54278.5	54508.2	53646.6
56462	55430.2	57199.94	55530.5	56039.4	54702.9	56567.6	56171.6	56015.6	55216.4	55535.8
57976	55649.8	56313.11	56206.9	55434.7	54892.8	56192.2	55902.7	56263.6	55566.1	56928.2
57930	56654.4	57872.39	57369.6	58269	55501.2	56901.2	56574.6	57591.5	56468.4	57810.5
55748	57159.9	57853.95	57775.3	57546.6	56517.8	57313.8	56717.9	57859	57165.4	58141
56071	57115.7	56787.94	56875	57289.3	57332.7	57030	56692.5	56863.6	56906.5	56457.9
56978	56273.4	57608.18	56283.1	56847.7	56357.4	57235.6	56344.1	56190.2	56295	56074
54450	56254.6	56450.19	56784	56583.8	55697.2	57358.2	56444.5	56791.6	56427.7	57002.6

Table 14
Test dataset (Z_{test}) for 2010.

Test	a	b	c	d	e	f	g	h	i	j
54112	55002.7	54570.12	55544.2	55626.9	55155.3	55481.2	54994	54980.4	55435.2	55263
54558	54175.8	55021.08	54683.5	55289.4	54194.5	55026.7	54385.5	54296.7	54655	54275
52257	54000.3	54351.85	54665.1	53843.5	53601.7	54938.6	54223.6	54475.5	54457.1	54687.1
54104	52585.6	52194.94	53381.9	54235.4	52911.8	53241.6	52660.6	52683.5	53350.1	52985.3
54498	53165.3	54032.2	53818.7	52800	52716.4	54286.2	53144.3	53822.1	53560.2	53850.2
55234	53692.7	54394.35	54294.3	54554.3	53315.4	54674.9	53360.7	54344.7	53956.1	54691.7
54385	54329.8	56158.79	54798.3	55279.5	53932.1	55447.3	53879.2	55024	54522.4	55152.8
RMSE	1221.17	1337.67	1189.41	1165.54	1282.34	1282.58	1328.60	1159.91	1177.19	1187.123
MAPE	0.020	0.021	0.018	0.019	0.021	0.019	0.022	0.019	0.018	0.018

Table 15
Forecasting results and RMSE/MAPE values of existing methods and MFF_{best}.

Test	ARIMA	ES	MLP-ANN	FF	FTS-N	R-T1FF	MFF_{best}
54112	54450	54449.936	54397.950	53543.007	55431.528	52984.810	55207.13
54558	54112	54111.607	54057.710	53788.681	54495.642	54419.000	54686.15
52257	54558	54558.150	54507.230	54475.042	54629.554	53166.650	54358.55
54104	52257	52257.121	52274.930	51697.376	53634.931	51463.420	53046.54
54498	54104	54103.459	54049.690	54377.843	53584.444	54183.150	53577.29
55234	54498	54497.941	54446.490	54445.135	53960.884	54071.150	54205.09
54385	55234	55233.664	55194.750	55221.339	54360.633	55612.920	54989.81
RMSE	1221.06	1221.09	1208.2	1360.85	1198.22	1311.99	1135.32*
MAPE	0.0183	0.018	0.018	0.020	0.017*	0.020	0.018

Table 16
Training dataset (Z_{train}) for 2011.

Test	a	b	c	d	e	f	g	h	i	j
67260	67488.8	67664.6	67506.8	67118.3	67485.4	67862	67500.5	67782.3	67820.8	67553.7
65643	66944.7	67173.5	66922.2	66669.3	66957.9	67384.6	66803.8	67268.4	67314.6	66610.2
66535	65292.9	66070.5	65300.2	66316.6	65871.7	66432.9	65378	66282.9	66738.1	66147.6
64585	66193.5	65714.5	66207.1	64761.7	64705.4	66029.4	65835.9	65903.6	65721.5	65513.3
65418	64250.2	65068.7	64244.6	65036.5	65314.7	65326.4	64415.3	65195.1	65393.3	64185.8
65385	65073.5	64589.6	65066.1	63952.1	65160.1	64957.5	65265.3	64819.6	64517.4	64940.6
63733	65045.3	64822.4	65044.1	63921.9	65215.8	64941.2	64881	64859.6	64282.4	63355.5
63299	63401.6	64115.9	63391.6	64487.2	63266.3	64317.2	64224.4	64208.1	64307.7	64101.4

The minimum RMSE value is obtained when $m = 2.5$ and $c = 3$ for Z_{train}. The best function (MFF_{best}) is determined as the first function that is given in Eq. (27).

$$MFF_{best} = MFF_1 = 0.04*a+0.14*b+\cdots+0.08*i+0.11*j \quad (27)$$

We are able to say that the performance of the proposed method is better than all existing forecasting methods in terms of RMSE value in Table 15.

3.2.3. 2011

There are 106 observations in BIST100 dataset for 2011 and the last 15 observations are taken as the test dataset. Using R-T1FF with different parameter specifications, 10 models are obtained. The first 8 observations of the 10 models are used as the training and the rest of the observations are use as the test dataset. Training and test sets are given in Tables 16–17, respectively.

The minimum RMSE value is obtained when $m = 2.6$ and $c = 4$. The weights of the models in functions are obtained by using the degree of membership values of the models in clusters for Z_{train}. The best function (MFF_{best}) is determined as the third meta fuzzy function and given in Eq. (28).

$$MFF_{best} = MFF_3 = 0.06*b+0.51*d+\cdots+0.09*i+0.18*j \quad (28)$$

Comparing MFF_{best} with the existing methods in Table 18, it is obvious that the best and second best forecasts are obtained from the proposed method in terms of RMSE and MAPE values, respectively.

3.2.4. 2012

The number of observations is 106 in BIST100 dataset for 2012. The forecasting results of 10 models of R-T1FFs method is given in Tables 19–20 as training and test datasets, respectively.

The minimum RMSE value is obtained when $m = 2$ and $c = 3$. MFF_{best} for 2012 is given in Eq. (29).

$$MFF_{best} = MFF_3 = 0.04*i+0.96*j \quad (29)$$

Comparing MFF_{best} with the existing methods in Table 21, it is obvious that the best and second best forecasts are obtained from the proposed method in terms of MAPE and RMSE values, respectively.

Table 17
Test dataset (Z_{test}) for 2011.

Test	a	b	c	d	e	f	g	h	i	j
63210	62968.8	63220.3	62965.1	63201	62989.6	63574.9	62960.3	63418.5	63777.3	63427.7
64561	62879.4	62686.5	62871.2	62430.2	62947.4	63099	63314.2	63018.6	62949.9	62495
63609	64242.4	62754	64326.7	63591.5	64247.5	63438.4	64000	64042.1	63697	63443.6
63755	63312.3	63802.2	63457	63183.2	63221.7	63792.7	62856.5	63681	64253.1	63557.2
62407	63448.5	62946.6	63572.1	63351.6	63079.2	63255.5	62789	63642.4	64630.3	63538.2
61492	62182.5	63391.2	62616.6	62117	62252.4	62962.3	62934.6	62161	62596.5	62527.2
63046	61209.2	62083.6	61411.3	61396.2	61180.1	61326.9	62461.2	61357.6	63358.2	61181.7
RMSE	1079.77	1022.34	1091.57	954.668	907.421	1043.02	918.301	987.321	1031.13	991.453
MAPE	0.0148	0.0140	0.0156	0.0134	0.0142	0.0138	0.0128	0.0132	0.0145	0.0159

Table 18
Forecasting results and RMSE/MAPE values of existing methods and MFF_{best}.

Test	ARIMA	ES	MLP-ANN	FF	FTS-N	R-T1FF	MFF_{best}
63210	63299	63298.61	63615.31	63500.67	63120.65	62597.67	63306.5
64561	63210	63209.89	63782.23	63121.86	62925.94	62356.36	62872.82
63609	64561	64561.3	64968.54	64690.38	63661.87	63079.91	63366.36
63755	63609	63609.38	63569.83	63565.97	63571.09	63262.51	63161.82
62407	63755	63755.34	64074.42	64245.59	63339.12	62956.39	63319.68
61492	62407	62407.52	62966.45	62360.53	62617.5	62324.37	62680.64
63046	61492	61491.78	62465.25	61782.2	61648.56	61212.24	62262.08
RMSE	1057.62	1057.84	1065.36	1139.72	986.08	1202.09	935.82
MAPE	0.0144	0.0144	0.0147	0.0158	0.0123	0.0159	0.0125

Table 19
Training dataset (Z_{train}) for 2012.

Test	a	b	c	d	e	f	g	h	i	j
58873	58571.8	58575.5	58375.1	58201.8	58338.4	58154.4	58719.7	58705.3	58547.4	58250.8
57854	58645.8	58653.2	58757.9	58196.4	58406.5	58258.9	58774.4	58778.3	58565.7	57440.3
57453	57630.5	57633.6	57495.1	57726.2	57777	57687.9	57761.6	57756.8	57677.1	57754.1
58101	57235.6	57224.9	57299.3	57387.5	57354	57193.3	57377.1	57356.7	57262.9	58171
57331	57899.7	57832.4	57846.7	57535.7	57614.9	57395.7	58087.9	58001.9	57751	57535.4
56936	57137.4	57057.3	57176.5	57193.6	57223.2	57090.3	57330.1	57233.4	57137.4	56661.6
56540	56724.5	56700.5	56653	56848.3	56807.6	56649.1	56870.2	56840.2	56731.9	57317.2
57079	56322.1	56322.9	56325.9	56474.9	56385.6	56233	56450.3	56445.4	56323.6	56831

Table 20
Test dataset (Z_{test}) for 2012.

Test	a	b	c	d	e	f	g	h	i	j
55734	56852.8	56871.7	56786.3	56522.6	56599.4	56398.3	56968.1	56983.9	56741.9	56511
54917	55519.1	55522.1	55527.7	55857.5	55734.9	55671	55646.4	55641	55573	55683.4
54810	54702.5	54712.4	54644.7	55234	54955.1	54859.1	54820.8	54825.9	54768.6	55166.9
54844	54612.6	54571.5	54646.7	54889.1	54653	54479.8	54783.6	54718.1	54593.9	54454.5
55450	54682.3	54529.8	54757.1	54746.7	54591.6	54428.9	54981.8	54749.7	54594.5	54635.1
55125	55288.7	55124.1	55369.1	55029.4	54950.9	54803	55613.4	55353.9	55091.7	55172
55099	54979.2	54774.2	55121.9	55046.8	55003.1	54914.4	55365	55028.2	54936.7	54867.2
RMSE	574.433	620.671	545.992	559.922	567.165	499.324	607.959	615.310	569.436	465.417
MAPE	0.0080	0.0087	0.0077	0.0079	0.0081	0.0072	0.0084	0.0080	0.0078	0.0074

Table 21
Forecasting results and RMSE/MAPE values of existing methods and MFF_{best}.

Test	ARIMA	ES	MLP-ANN	FF	FTS-N	R-T1FF	MFF_{best}
55734	57079	57079.46	57421.46	57781.04	56717.86	56461.83	56472.86
54917	55734	55734.36	56014.45	56433.76	56055.82	55329.15	55338.75
54810	54917	54916.62	55218.22	55638.09	55140.42	54546.99	54555.71
54844	54810	54809.94	55143.92	55762.38	54823.27	54533.63	54536
55450	54844	54843.93	55185.19	55733.9	54811.12	54731.61	54726.21
55125	55450	55449.5	55823.22	56244.47	55193.66	55193.92	55189.89
55099	55125	55125.29	55452.13	55899.09	55181.59	55460.04	55439.44
RMSE	650.5663	650.7343	844.381	1194.874	631.8026	465.417*	467.1011
MAPE	0.0084	0.0084	0.01243	0.01945	0.0084	0.00739	0.00737*

3.3. Dow Jones datasets

There are 4 datasets for Dow Jones index from 2010 to 2013. To obtain 10 models from R-T1FF, the number of clusters was varied from 2 to 5, the AR lag was varied from 1 to 5, and the MA lag was varied from 1 to 2. The last 10 observations each Dow Jones datasets are selected as the test dataset (Y_{test}). The input matrix (Z) of MFFs are obtained from the 10 point forecasts of 10 models. The first 6 observations of Z is selected as the training matrix (Z_{train}) in order to determine the weights of the models in functions and MFF_{best}. The last 4 observations are left for test dataset (Z_{test}) to evaluate the performance of the proposed method with the existing methods; ANFISgrid, ANFISsub, MANFIS [32], T1FF [5].

Table 22
Training dataset (Z_{train}) for 2010.

Test	a	b	c	d	e	f	g	h	i	j
11491.91	11509.24	11496.92	11447.94	11508.6	11504.61	11501.48	11506.74	11542.65	11524.84	11461.46
11478.13	11496.21	11513.26	11495.37	11498.94	11497.21	11501.37	11511.26	11510.08	11510.12	11525.1
11533.16	11486.68	11499.99	11420.47	11485.82	11483.4	11487.25	11497.82	11564.27	11498.92	11469.43
11559.49	11552.89	11525.37	11538.89	11532.86	11530.86	11533.05	11538.34	11571.21	11545.27	11504.03
11573.49	11584.82	11559.2	11538.18	11560.06	11557.92	11559.64	11565.89	11526.91	11571.88	11532.97
11555.03	11621.4	11580.19	11567.66	11564.82	11560.63	11574.13	11581.22	11579.32	11573.67	11545.42

Table 23
Test dataset (Z_{test}) for 2010.

Test	a	b	c	d	e	f	g	h	i	j
11575.54	11587.86	11579.73	11529.32	11558.87	11556.17	11556.73	11565.74	11613.85	11569.1	11572.72
11585.38	11633.49	11587.79	11559.15	11565.57	11558.74	11572.06	11578.47	11647.2	11570.81	11552.09
11569.71	11661.2	11594.12	11577.85	11568.09	11566.77	11584.86	11591.76	11616.61	11577.46	11583.81
11577.51	11600.34	11595.19	11539.11	11583.51	11577.85	11569.66	11578.07	11635.35	11591.52	11585.33
RMSE	53.287	15.263	33.035	13.313	16.535	14.339	12.553	52.044	11.293	18.548
MAPE	0.00377	0.00105	0.00257	0.00095	0.00106	0.00119	0.00085	0.00442	0.00092	0.00125

Table 24
Weights of the models in functions and RMSE values of the functions for Z_{train}.

Model	MFF_1	MFF_2	MFF_3	MFF_4
a	0	0	0	0.987
b	0.165	0	0	0
c	0	0	0.476	0
d	0.167	0	0	0
e	0.165	0	0	0
f	0.167	0	0	0
g	0.167	0.001	0.001	0.002
h	0	0.992	0	0
i	0.165	0.004	0.001	0.009
j	0.005	0.003	0.522	0.002
RMSE	24.93*	35.00	46.44	34.81
MAPE	0.0020*	0.0028	0.0034	0.0024

Table 25
Forecasts of MFFs for 2010 and RMSE and MAPE values of the functions.

Model	MFF_1*	MFF_2	MFF_3	MFF_4
7	11564.4	11613.49	11552.04	11587.61
8	11572.12	11646.52	11555.49	11632.63
9	11580.52	11616.32	11580.97	11660.12
10	11582.6	11634.96	11563.32	11600.18
RMSE	10.52*	51.60	21.06	52.60
MAPE	0.0009*	0.0044	0.0017	0.0037

The number of clusters is varied from 2 to 5 and the fuzziness value is varied from 1.3 to 3 with the increase rate of 0.1 for MFFs. Although the detailed results are given for the dataset of 2010, only the input matrices and formal comparison results are given for 2011–2013.

3.3.1. 2010

For Dow Jones dataset in 2010, 10 forecasts of 10 models are used as the input matrix, Z. The first 6 observations of Z is used as training matrix (Z_{train}) and the last 4 observations are used as the test dataset (Z_{test}) and they are given in Tables 22 and 23, respectively.

The best MFFs are obtained when $m = 1.4$ and $c = 4$. The weight matrix that are calculated by FCM for Z_{train} is given in Table 24. The best function is determined from MAPE values of the MFFs for Z_{train}. In this case, MFF_1 is selected as MFF_{best}. Thus, the forecasts of the proposed methods is obtained by using MFF_{best}. MFF_{best} is given in Eq. (30) and the corresponding forecasting results and the results of other functions are given in Table 25.

$$MFF_{best} = MFF_1 = 0.165 * a + \cdots + 0.165 * i + 0.005 * j \tag{30}$$

Table 26
Forecasting results and RMSE/MAPE values of existing methods and MFF_{best}.

Test	ANFISgrid	ANFISsub	MANFIS	T1FF	MFF_{best}
11575.54	11571.59	11587.34	11563.86	11562.97	11564.4
11585.38	11601.37	11606.63	11584.40	11580.95	11572.12
11569.71	11591.92	11617.79	11594.25	11589.56	11580.52
11577.51	11546.67	11606.31	11578.56	11575.85	11582.6
RMSE	20.71	30.54	13.61	11.98	10.52
MAPE	0.0016	0.0024	0.0008	0.0008	0.0008

4 existing forecasting methods are compared with the proposed method in Table 26. It is obvious that the MFF_{best} of the proposed methods outperforms other methods in terms of both RMSE and MAPE values.

3.3.2. 2011

There are 251 observations in Dow Jones dataset for 2011 and the last 10 observations are taken as the test dataset (Y_{test}). Using Y_{train}, 10 models are obtained from R-T1FF with different parameter specifications. In order to obtain the input matrix (Z) of MFFs, 10 forecast are calculated for each model that are obtained from R-T1FFs. The first 6 forecasts of the 10 models are used as the training (Z_{train}) and the rest of the forecasts are used as the test (Z_{test}) datasets. Training and test sets are given in Tables 27–28, respectively.

The best MFFs are obtained when $m = 1.4$ and $c = 5$. The weights of the models in functions are obtained by using the degree of membership values of the models in clusters for Z_{train}. The best function (MFF_{best}) is determined as the fifth meta fuzzy function and given in Eq. (31).

$$MFF_{best} = MFF_5 = 0.85 * b + 0.143 * g + 0.003h + 0.01 * i + 0.03 * j \tag{31}$$

We are able to say that the performance of the proposed method is better than all existing forecasting methods in terms of RMSE and MAPE values in Table 29.

3.3.3. 2012

The 10 forecasts of the 10 models are used as the input matrix (Z) of MFFs. The input matrix is divided into two training (Z_{train}) and test (Z_{test}) sets. Tables 30–31 represents the training and test sets, respectively.

Training dataset is clustered by using FCM and the weights of the models that are calculated from the degrees of memberships are obtained for each function. The best set of meta fuzzy functions

Table 27
Training dataset (Z_{train}) for 2011.

Test	a	b	c	d	e	f	g	h	i	j
11866.39	11857	11855.71	11897.39	11850.31	11878.25	11882.41	11926.76	11972.49	11833.98	11875.74
11766.26	11854.57	11885.18	11892.79	11838.86	11860.4	11866.2	11907.15	11934.58	11855.61	11860.47
12103.58	11749.06	11949.04	11789.68	11718.03	11748.25	11758.53	11840.78	11854.75	11760.46	11752.51
12107.74	12092.35	12010.28	12129.22	12085.48	12176.49	12159.52	12017.95	12181.51	12084.2	12162.54
12169.65	12098.85	12423.39	12133.78	12094.52	12086.54	12103.93	12232.23	12183.75	12082.13	12088.14
12294	12165.39	12345.28	12196.03	12190.67	12193.46	12192.07	12211.43	12236.02	12125	12190.63

Table 28
Test dataset (Z_{test}) for 2011.

Test	a	b	c	d	e	f	g	h	i	j
12291.35	12283.01	12349.38	12320.18	12265.27	12302.62	12302.62	12292.61	12375.88	12276.75	12296.55
12151.41	12266.55	12295.66	12316.39	12218.12	12280.77	12286.05	12315.51	12387.26	12300.29	12282.49
12287.04	12135.2	12271.84	12176.54	12118.61	12132.35	12143.89	12185.22	12235.38	12141.79	12141.97
12217.56	12258.35	12254.01	12311.6	12194.92	12270.37	12260.35	12250.39	12387.7	12302.51	12259.55
RMSE	97.53	80.21	110.80	92.21	104.38	100.72	97.95	153.61	112.57	100.02
MAPE	0.0065	0.0052	0.0082	0.0058	0.0071	0.0068	0.0061	0.0111	0.0081	0.0066

Table 29
Forecasting results and RMSE/MAPE values of existing methods and MFF_{best}.

Test	ANFISgrid	ANFISsub	MANFIS	T1FF	MFF_{best}
12291.35	12268.61	12329.13	12285.16	12271.36	12341.09
12151.41	12285.36	12296.95	12319.05	12285.72	12298.72
12287.04	12115.69	12149.06	12218.30	12150.31	12258.83
12217.56	12259.14	12290.89	12274.94	12261.81	12253.92
RMSE	111.30	108.43	95.08	98.86	81.07
MAPE	0.0076	0.0081	0.0061	0.0069	0.0054

Table 32
Forecasting results and RMSE/MAPE values of existing methods and MFF_{best}.

Test	ANFISgrid	ANFISsub	MANFIS	T1FF	MFF_{best}
13114.59	13171.73	13119.31	13152.26	13111.70	13074.81
13096.31	13183.93	13091.44	13123.75	13091.97	13090.32
12938.11	12982.05	13070.81	13104.55	13082.15	13029.86
13104.14	12947.44	12929.84	12966.91	12956.00	12949.25
RMSE	96.73	109.59	110.35	103.34	92.23
MAPE	0.0066	0.0061	0.0071	0.0057	0.0056

are obtained when $m = 3$ and $c = 5$. The best meta fuzzy function is detected as the third function and it is given in Eq. (32).

$$MFF_{best} = MFF_3 = 0.85 * b + 0.143 * g + 0.003h + 0.01 * i + 0.03 * j \quad (32)$$

Table 32 is given to compare the results of the existing methods and the proposed method. The table shows that the proposed method outperforms the other methods in terms of both RMSE and MAPE values.

3.3.4. 2013

For 2013, the training and test datasets for MFFs are given in Tables 33–34, respectively. The best set of MFFs are obtained when $m = 2$ and $c = 4$. The best function is determined as the fourth function for Z_{train}. Thus the MFF_{best} is the fourth function of the MFFs and given in Eq. (33).

$$MFF_{best} = MFF_4 = 0.39 * c + 0.304 * g + 0.278 * h + 0.015 * i + 0.013 * j \quad (33)$$

Table 35 is given to compare the results of the existing methods and the proposed method. The table shows that the proposed method has better forecasting performances in terms of both RMSE and MAPE values.

4. Conclusions

A naive approach based on FCM is introduced to aggregate the methods in the paper. MFFs are able to cluster the methods in functions. For example, while the methods that perform better for a dataset is collected in a function, the methods that perform worse is collected in a different function with certain degrees of memberships values. The advantages of the MFFs are listed below.

- MFFs is the first introduced approach that aims to aggregate methods into functions. MFFs are introduced with an assumption that each method has its own understanding of a

Table 30
Training dataset (Z_{train}) for 2012.

Test	a	b	c	d	e	f	g	h	i	j
13235.39	13056.98	13176.11	13038.39	13073.12	13113.15	13209.58	13235.87	13148.85	13073.29	13042.41
13350.96	13224.84	13246.35	13233.24	13228.72	13229.49	13345.16	13284.5	13224.6	13161.99	13147.94
13251.97	13278.11	13383.79	13252.59	13290.75	13346.29	13438.05	13308.66	13340.09	13276.86	13330.3
13311.72	13246.44	13274.32	13257.43	13255.6	13223.93	13308.26	13235.23	13283.47	13250.49	13274.17
13190.84	13239.1	13338.54	13209.6	13247.12	13301.04	13367.06	13427.85	13304.08	13219.84	13175.33
13139.08	13168.5	13207.68	13190.34	13178.6	13157.27	13262.86	13198.67	13206.96	13191.14	13218.59

Table 31
Test dataset (Z_{test}) for 2012.

Test	a	b	c	d	e	f	g	h	i	j
13114.59	13071.78	13163.95	13035.07	13077.51	13114.57	13218.85	13262.62	13153.67	13074.91	13152.67
13096.31	13086.45	13131.89	13115.85	13098.23	13093.7	13189.57	13185.82	13115.04	13077.62	12982.81
12938.11	13039.59	13123.11	12994.85	13044.86	13076.02	13163.45	13142.86	13103.29	13043.67	13081.43
13104.14	12899.95	12952.87	12937.57	12911.22	12897.47	13005.45	13001.49	12965.96	12922.84	13038.9
RMSE	116.11	123.30	97.04	111.79	124.24	141.50	143.51	109.84	107.16	98.91
MAPE	0.0069	0.0081	0.0062	0.0065	0.0067	0.0100	0.0104	0.0069	0.0066	0.0069

Table 33
Training dataset (Z_{train}) for 2013.

Test	a	b	c	d	e	f	g	h	i	j
15875.26	15924.92	15903.44	15919.36	15935.81	15856.47	15989.79	15908.92	15982.18	15832.66	15858.63
16167.97	15920.97	15896.86	15920.98	15898.55	15840.76	15985.11	15949.97	15926.9	15881.37	15887.72
16179.08	16200.91	16184.55	16218.42	16206.11	16146.1	16280.69	16140.39	16242.17	16043.13	16095.08
16221.14	16210.18	16194.1	16230.46	16210.97	16163.67	16291.27	16241.09	16243.89	16170.52	16178.51
16294.61	16257.8	16237.63	16272.24	16258.9	16200.72	16332.8	16277.16	16292	16207.91	16220.33
16357.55	16331.07	16311.87	16346.56	16326.83	16269.35	16407.64	16344.89	16361.56	16266.5	16286.06

Table 34
Test dataset (Z_{test}) for 2013.

Test	a	b	c	d	e	f	g	h	i	j
16479.88	16390.09	16374.52	16410.98	16389.69	16333.34	16472.29	16415.27	16424.57	16332.66	16352.38
16478.41	16511.81	16496.75	16534.34	16511.73	16458.25	16595.82	16518.32	16547.13	16427.8	16457.44
16504.29	16511.01	16495.34	16533.47	16509.64	16456.77	16594.71	16562.95	16545	16481.17	16492.45
16576.66	16536.43	16521.29	16560	16534.83	16481.1	16621.23	16581.48	16570.57	16498.68	16514.53
RMSE	52.06	60.38	47.45	52.49	91.20	77.47	48.04	48.67	87.82	71.93
MAPE	0.0026	0.0028	0.0026	0.0026	0.0047	0.0039	0.0025	0.0026	0.0045	0.0034

Table 35
Forecasting results and RMSE/MAPE values of existing methods and MFF_{best}.

Test	ANFISgrid	ANFISsub	MANFIS	T1FF	MFF_{best}
16479.88	16236.68	16281.99	16377.62	16390.06	16414.16
16478.41	16353.20	16365.03	16495.93	16507.48	16530.48
16504.29	16214.60	16345.41	16503.17	16510.77	16544.35
16576.66	16256.59	16357.09	16527.16	16536.35	16567.99
RMSE	255.53	177.11	57.48	51.43	46.66
MAPE	0.0148	0.0104	0.0026	0.0025	0.0025

dataset. For example, for a specific purpose like forecasting or predicting, there are many methods in literature mainly using the same arguments with different modifications. Therefore, our assumption is that each method has more or less knowledge for a given dataset.

- One does not need to have a deep understanding of fuzzy set theory to implement or understand the proposed method. The only need for implementing the proposed method is to understand the FCM algorithm.
- MFFs guarantees, at least, the best performance of the methods that are used. In another word, MFFs are also able to search for the best method among many. However, MFFs usually increase the performance of the related methods by aggregating them into functions in terms of some evaluation criteria.

Although the aim of the study is to aggregate the methods for a specific purpose, the models of a method with different parameter specifications are aggregated as an application in the paper. 9 real world datasets were chosen for the evaluation. Results showed that MFFs are able to increase the performance of R-T1FFs. For all nine datasets, the best performances for forecasting in terms of both RMSE and MAPE values were obtained from the MFFs approach. Besides, comparing the results of the proposed method with other alternative methods in literature, it can be said that MFFs based R-T1FFs outperforms most of them in terms of both RMSE and MAPE. Aggregating methods for forecasting and employing the possibilistic FCM are left for the future work. Besides, aggregation of regression methods and indices are another future works.

Acknowledgment

I would like to thank to the handling editor Prof. Dr. Oscar Castillo and the reviewers for their helpful comments and suggestions. And, also, I would like to thank to the people of Forecast Research Laboratory of Giresun University for their comments and suggestions.

References

[1] G.V. Glass, Primary, secondary, and meta-analysis of research, Educ. Researcher 5 (10) (1976) 3–8, http://dx.doi.org/10.3102/0013189X005010003.
[2] R. DerSimonian, N. Laird, Meta-analysis in clinical trials, Controlled Clin. Trials 7 (3) (1986) 177–188, http://dx.doi.org/10.1016/0197-2456(86)90046-2.
[3] J.C. Bezdek, R. Ehrlich, W. Full, FCM: The fuzzy c-means clustering algorithm, Comput. Geosci. 10 (2–3) (1984) 191–203, http://dx.doi.org/10.1007/s10898-007-9149-x.
[4] N. Tak, A.A. Evren, M. Tez, E. Egrioglu, Recurrent type-1 fuzzy functions approach for time series forecasting, Appl. Intell. 48 (1) (2018) 68–77, http://dx.doi.org/10.1007/s10489-017-0962-8.
[5] I.B. Turksen, Fuzzy functions with LSE, Appl. Soft Comput. 8 (3) (2008) 1178–1188, http://dx.doi.org/10.1016/j.asoc.2007.12.004.
[6] S. Beyhan, M. Alci, Fuzzy functions based ARX model and new fuzzy basis function models for nonlinear system identification, Appl. Soft Comput. 10 (2010) 439–444, http://dx.doi.org/10.1016/j.asoc.2009.08.015.
[7] C.H. Aladag, I.B. Turksen, A.Z. Dalar, E. Egrioglu, U. Yolcu, Application of type-1 fuzzy functions approach for time series forecasting, Turkish J. Fuzzy Syst. 5 (1) (2014) 1–9.
[8] R. Krishnapuram, J.M. Keller, A possibilistic approach to clustering, IEEE Trans. Fuzzy Syst. 1 (2) (1993) 98–110, http://dx.doi.org/10.1109/91.227387.
[9] N.R. Pal, K. Pal, J.M. Keller, J.C. Bezdek, A possibilistic fuzzy c-means clustering algorithm, IEEE Trans. Fuzzy Syst. 13 (4) (2005) 517–530, http://dx.doi.org/10.1109/TFUZZ.2004.840099.
[10] E. Rubio, O. Castillo, F. Valdez, P. Melin, C.I. Gonzalez, G. Martinez, An extension of the fuzzy possibilistic clustering algorithm using type-2 fuzzy logic techniques, Adv. Fuzzy Syst. 2017 (2017) 1–23, http://dx.doi.org/10.1155/2017/7094046.
[11] Q. Song, B.S. Chissom, Forecasting enrollments with fuzzy time series – Part I, Fuzzy Sets and Systems 54 (1) (1993) 1–9, http://dx.doi.org/10.1016/0165-0114(93)90355-L.
[12] Q. Song, B.S. Chissom, Forecasting enrollments with fuzzy time series – Part II, Fuzzy Sets and Systems 62 (1) (1994) 1–8, http://dx.doi.org/10.1016/0165-0114(94)90067-1.
[13] S.M. Chen, Forecasting enrollments based on fuzzy time series, Fuzzy Sets and Systems 81 (3) (1996) 311–319, http://dx.doi.org/10.1016/0165-0114(95)00220-0.
[14] D.W. Chen, J.P. Zhang, Time series prediction based on ensemble ANFIS, in: Machine Learning and Cybernetics, Proceedings of 2005 International Conference, vol. 6, 2005, pp. 3552–3556, http://dx.doi.org/10.1109/ICMLC.2005.1527557.
[15] C. Gupta, A. Jain, D.K. Tayal, O. Castillo, Forecasting low dimensional numerical data using an improved method based on automatic clustering, fuzzy relationships and differential evolution, Eng. Appl. Artif. Intell. 71 (2018) 175–189, http://dx.doi.org/10.1016/j.engappai.2018.02.015.
[16] C.H. Aladag, U. Yolcu, E. Egrioglu, A.Z. Dalar, A new time invariant fuzzy time series forecasting method based on particle swarm optimization, Appl. Soft Comput. 12 (10) (2012) 3291–3299, http://dx.doi.org/10.1016/j.asoc.2012.05.002.
[17] S. Makridakis, Why combining works? Int. J. Forecast. 5 (4) (1989) 601–603, http://dx.doi.org/10.1016/0169-2070(89)90017-4.
[18] J.M. Bates, C.W. Granger, The combination of forecasts, J. Oper. Res. Soc. 20 (4) (1969) 451–468, http://dx.doi.org/10.1057/jors.1969.103.
[19] C.W. Granger, R. Ramanathan, Improved methods of combining forecasts, J. Forecast. 3 (2) (1984) 197–204, http://dx.doi.org/10.1002/for.3980030207.

[20] D.W. Bunn, A Bayesian approach to the linear combination of forecasts, J. Oper. Res. Soc. 26 (2) (1975) 325–329, http://dx.doi.org/10.1057/jors.1975.67.
[21] P. Newbold, C.W. Granger, Experience with forecasting univariate time series and the combination of forecasts, J. R. Stat. Soc. Ser. A (1974) 131–165, http://dx.doi.org/10.2307/2344546.
[22] H. Zou, Y. Yang, Combining time series models for forecasting, Int. J. Forecast. 20 (1) (2004) 69–84, http://dx.doi.org/10.1016/S0169-2070(03)00004-9.
[23] C.H. Aladag, E. Egrioglu, U. Yolcu, Forecast combination by using artificial neural networks, Neural Process. Lett. 32 (3) (2010) 269–276, http://dx.doi.org/10.1007/s11063-010-9156-7.
[24] N.R. Pal, J.C. Bezdek, On cluster validity for the fuzzy c-means model, IEEE Trans. Fuzzy Syst. 3 (3) (1995) 370–379, http://dx.doi.org/10.1109/91.413225.
[25] I. Ozkan, I.B. Turksen, Upper and lower values for the level of fuzziness in FCM, Inform. Sci. 177 (23) (2007) 5143–5152, http://dx.doi.org/10.1016/j.ins.2007.06.028.
[26] K.P. Chan, Y.S. Cheung, Clustering of clusters, Pattern Recognit. 25 (2) (1992) 211–217, http://dx.doi.org/10.1016/0031-3203(92)90102-O.
[27] J.C. Bezdek, A physical interpretation of fuzzy ISODATA, Read. Fuzzy Sets Intell. Syst. (1993) 615–616, http://dx.doi.org/10.1016/B978-1-4832-1450-4.50065-1.
[28] G.J. Janacek, Practical Time Series, Oxford University Press Inc., New York, 2001.
[29] BIST100, Istanbul stock exchange index dataset, 2015. http://www.borsaistanbul.com/veriler/gecmise-donuk-veri-satisi. (Accessed 5 November 2015).
[30] G.E. Box, G.M. Jenkins, G.C. Reinsel, G.M. Ljung, Time Series Analysis: Forecasting and Control, John Wiley and Sons, 2015.
[31] J.S. Jang, ANFIS: Adaptive-network-based fuzzy inference systems, IEEE Trans. Syst. Man Cybern. 23 (3) (1993) 665–685, http://dx.doi.org/10.1109/21.256541.
[32] E. Egrioglu, C.H. Aladag, U. Yolcu, E. Bas, A new adaptive network based fuzzy inference system for time series forecasting, Aloy J. Soft Comput. Appl. 2 (2014) 25–32.
[33] B. Sarica, E. Egrioglu, B. Asikgil, A new hybrid method for time series forecasting: Aranfis, Neural Comput. Appl. 29 (3) (2018) 749–760, http://dx.doi.org/10.1007/s00521-016-2475-5.
[34] E.S. Gardner, Exponential smoothing: The state of the art, J Forecast. 4 (1) (1985) 1–28, http://dx.doi.org/10.1002/for.3980040103.
[35] E. Bas, E. Egrioglu, C.H. Aladag, U. Yolcu, Fuzzy-time-series network used to forecast linear and nonlinear time series, Appl. Intell. 43 (2) (2015) 343–355, http://dx.doi.org/10.1007/s10489-015-0647-0.