Can a signal propagate superluminal (v>c) in dispersive medium?

M. Emre Taşgın

Outline

- Experiment: superluminal (v>c) propagation.
- Reshaping due to gain/absorption
- A theoretical method to test if velocity is reliable?
- Answer: is superluminal?
- Acknowledgements.

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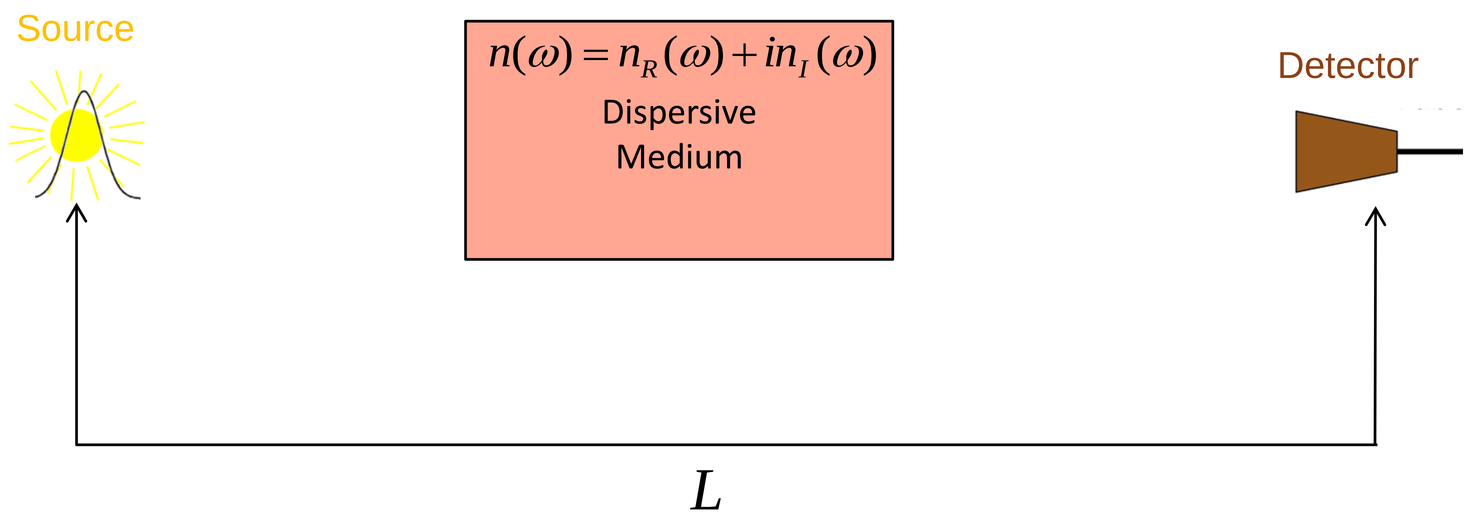
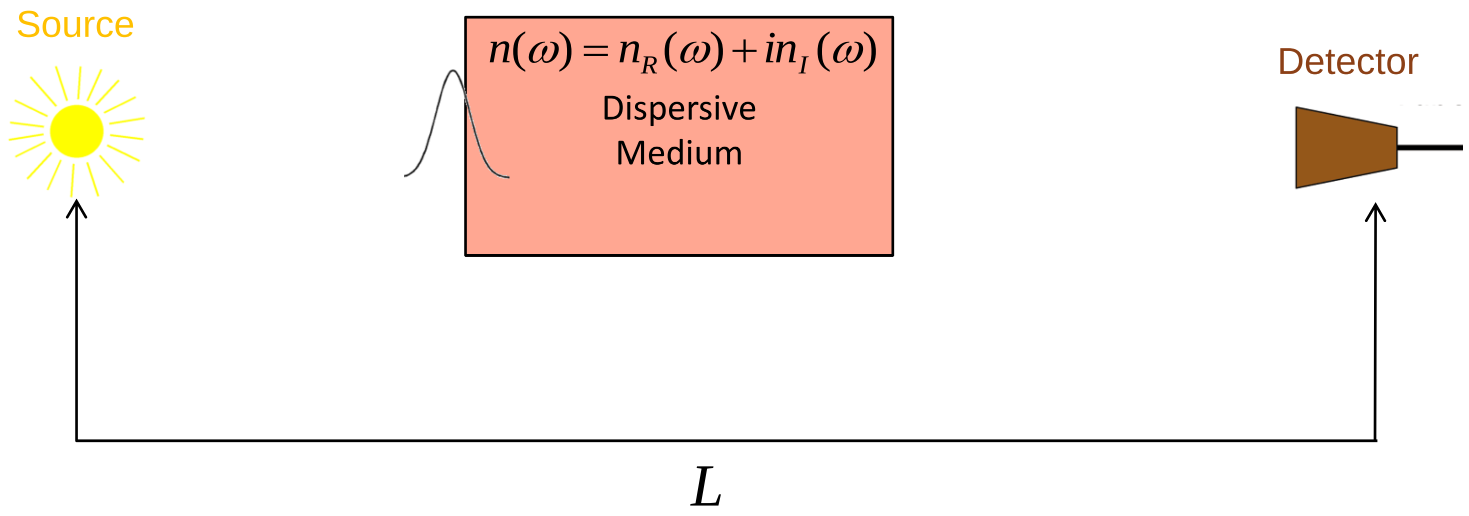
Experiment


Source
$n(\omega) = n_R(\omega) + i n_I(\omega)$
Dispersive
Medium
Detector
L

Experiment



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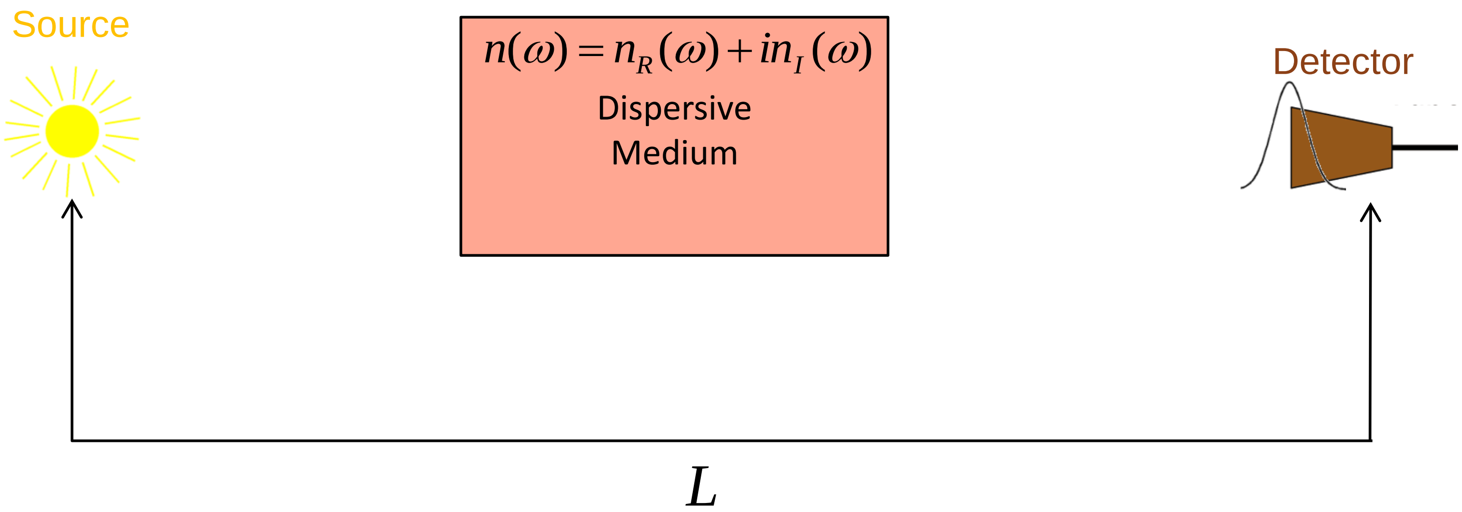
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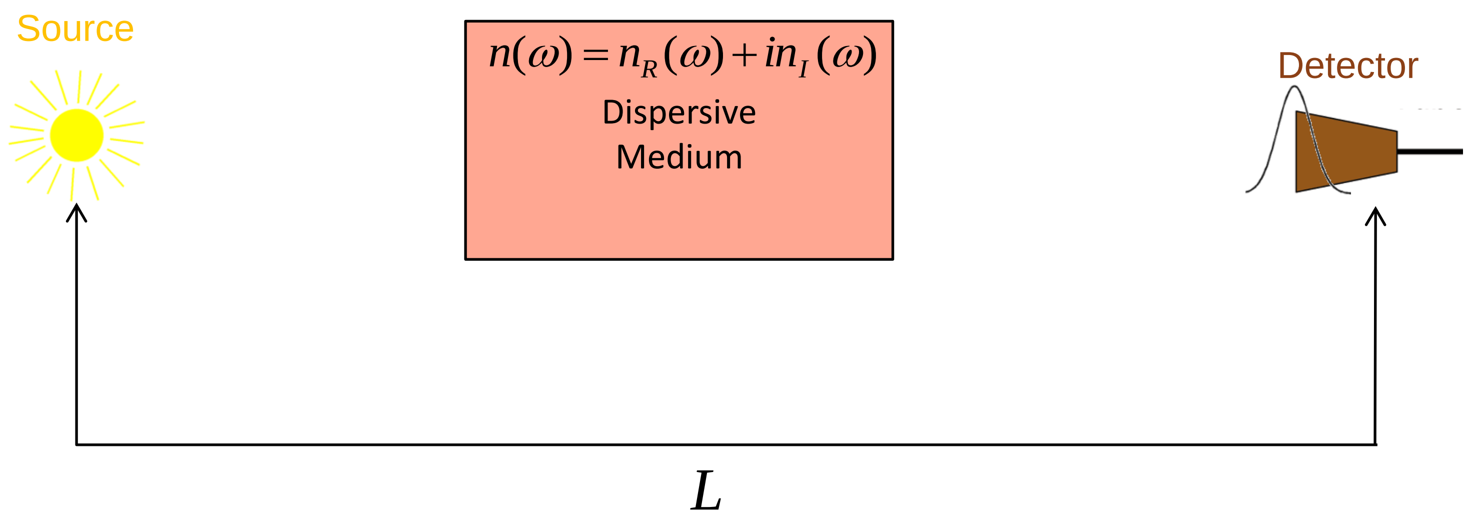
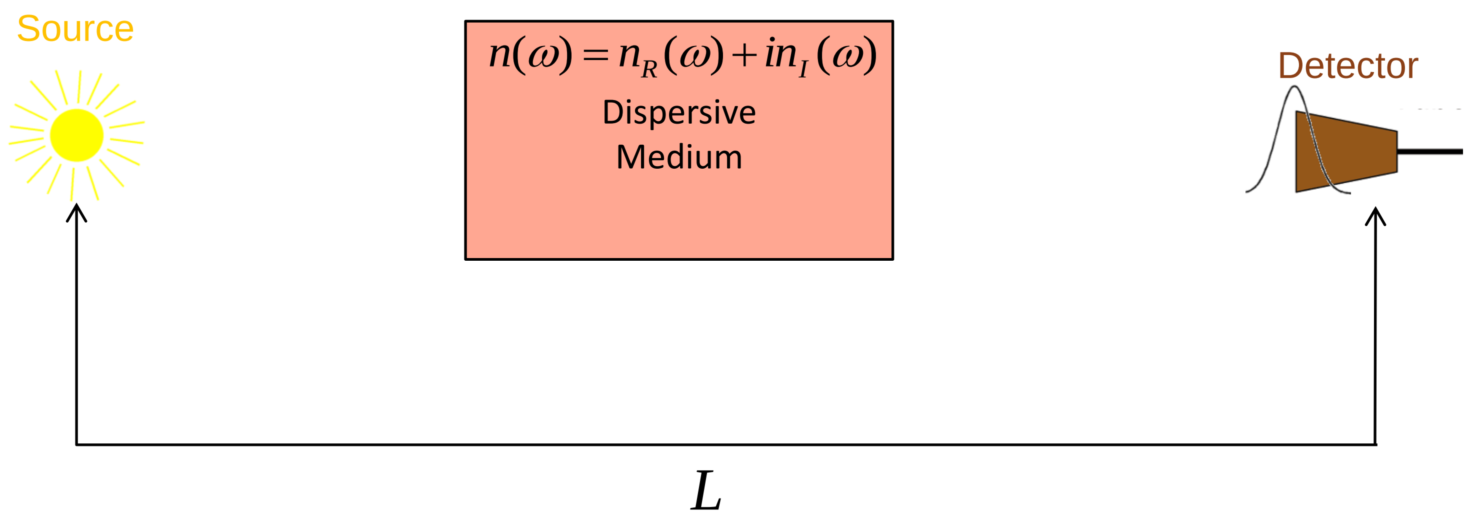


if travels with speed of light $\Rightarrow$ $\Delta t_0 = L / c$

Experiment

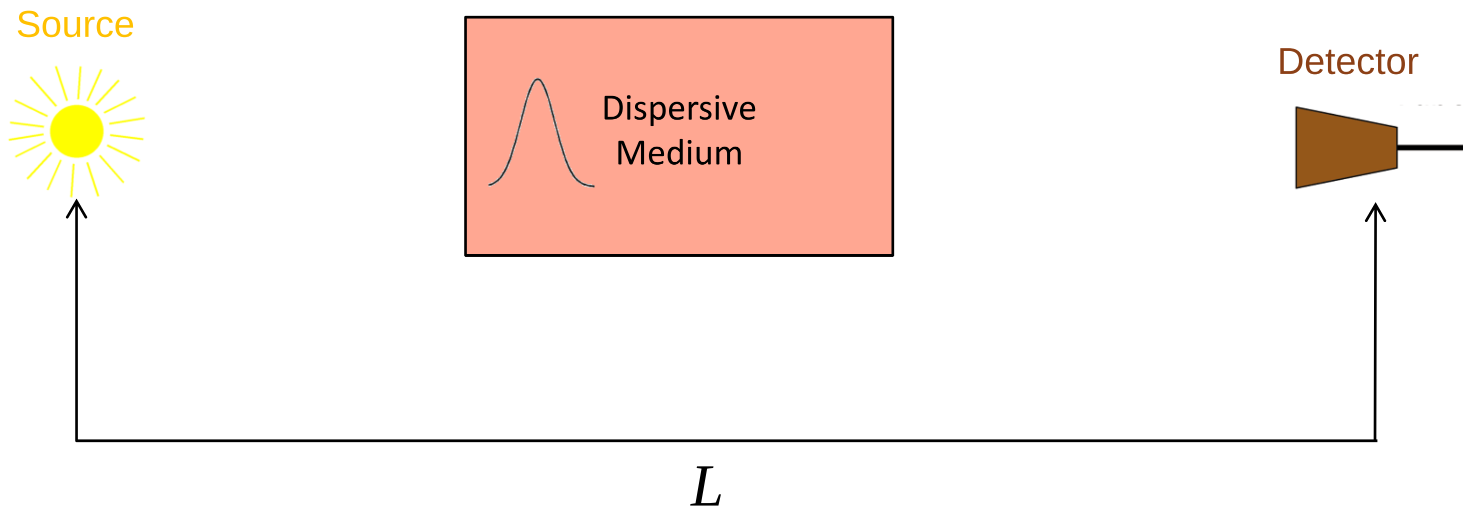


if travels with speed of light $\Rightarrow$ $\Delta t_0 = L / c$

superluminal propagation $\Leftarrow$ if $\Delta t < \Delta t_0$ [1]

[1] L. J.Wang, A. Kuzmich, and A. Dogariu, Nature (London) **406,** 277 (2000).

Problem!



Pulse displaces:

- ❑ Where to choose the reference point for displacement?
- ❑ Pulse also reshapes due to gain/absorption.

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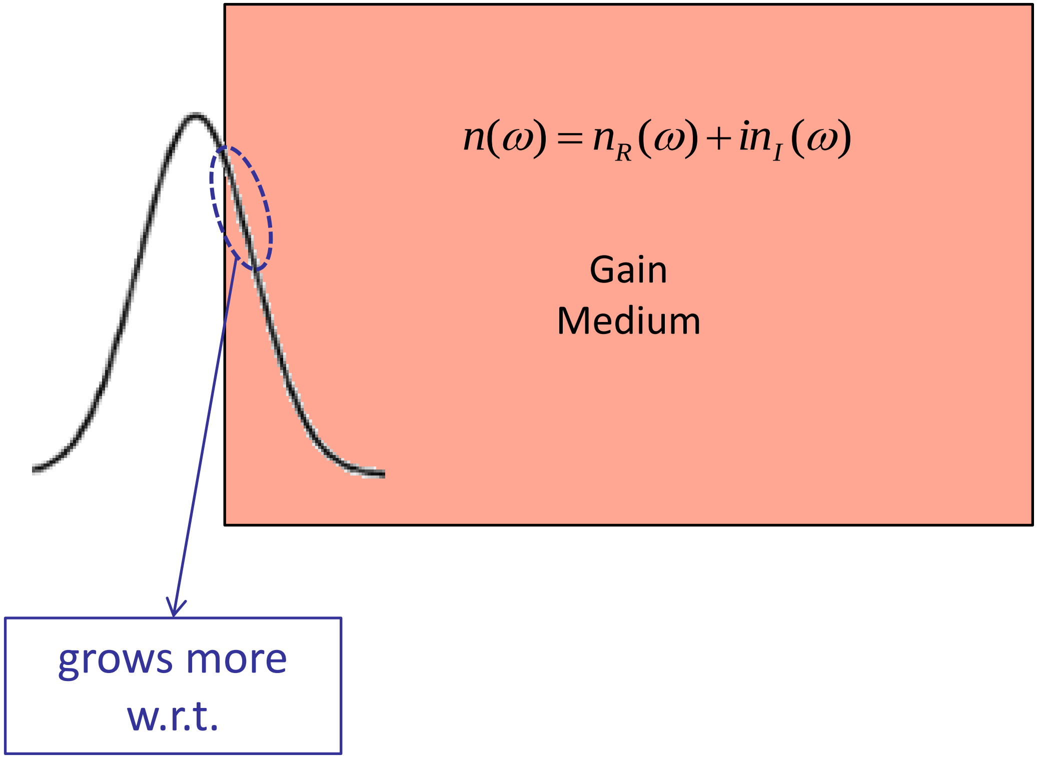
example for reshaping



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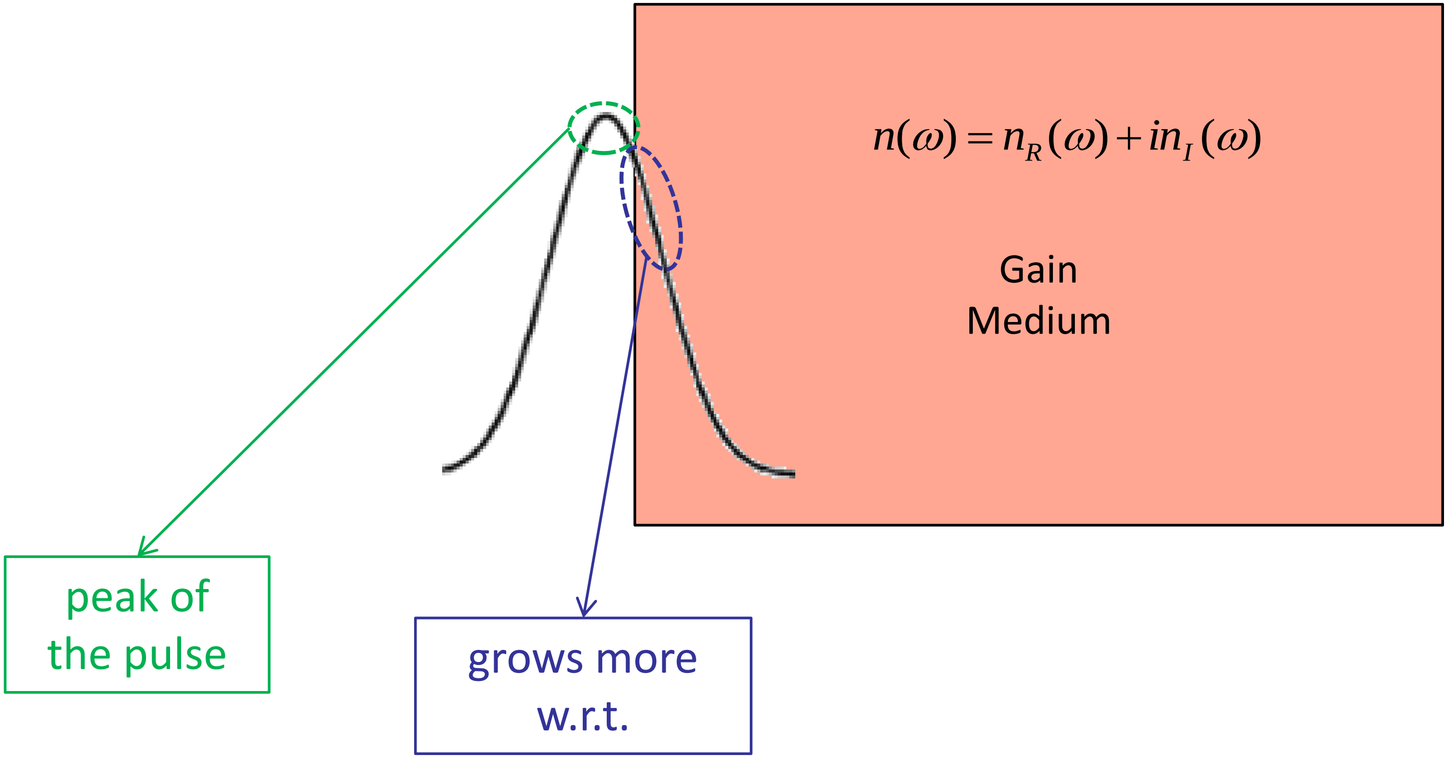

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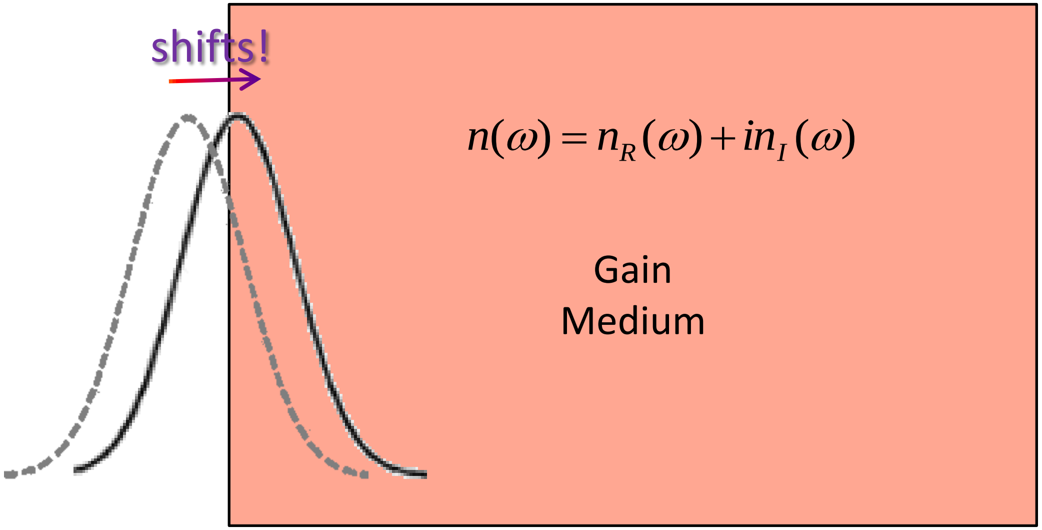
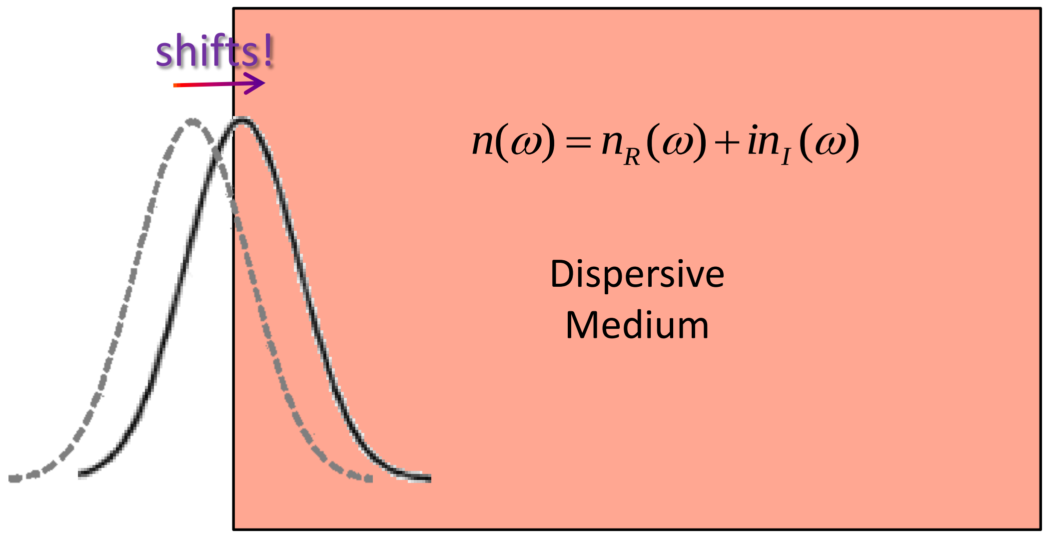
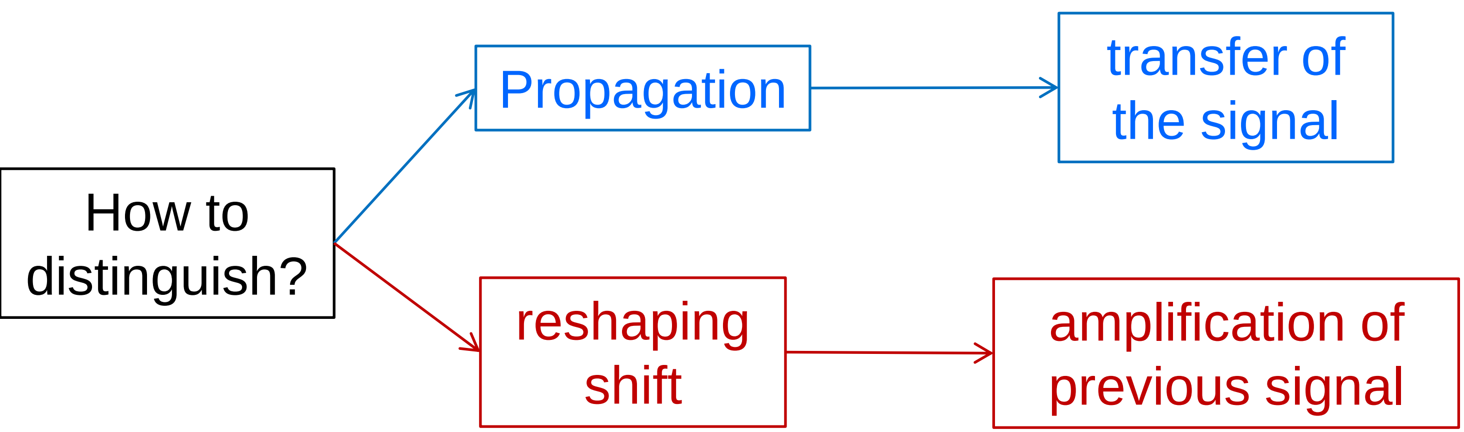


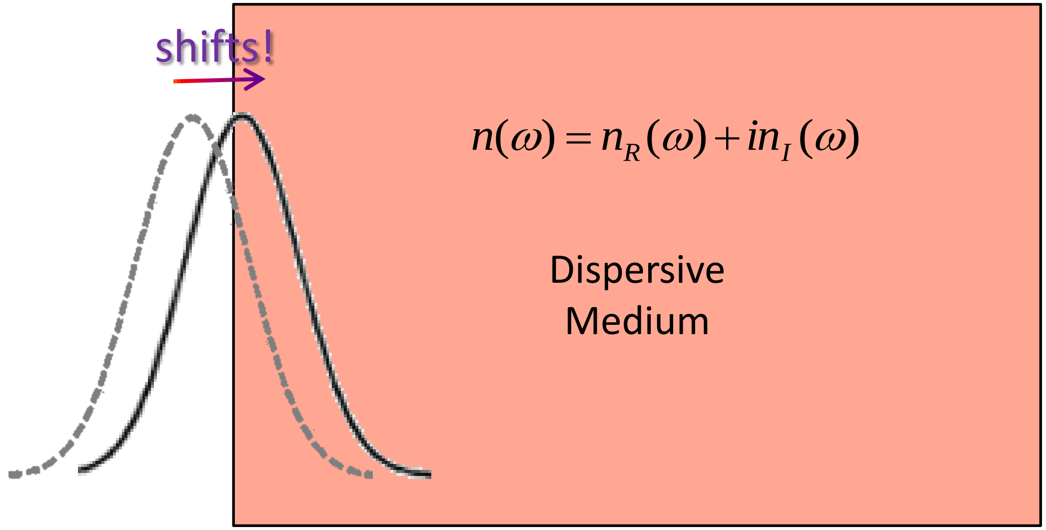
Pulse shifts right. → not due to propagation

Problem: to distinguish





experiments

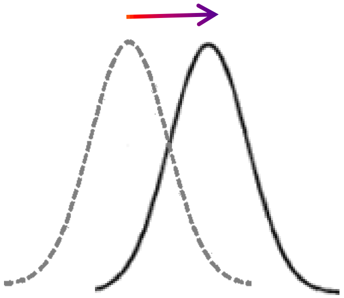


experiments detect → pulse peak → detect amplified pulse!

experiments detect → averaged pulse → detect amplified pulse!

Velocity definitions

- Displacement of the pulse peak



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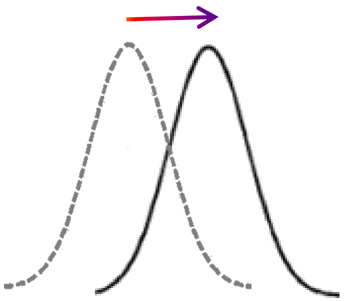

- ➢ Displacement of the pulse peak

- ➢ Energy/Poynting-vector averaged pulse center

$$\langle t \rangle_x = \frac{\int dt\, t\, S(x,t)}{\int dt S(x,t)}$$ [2]

$$\langle x \rangle_t = \frac{\int dx\, x\, S(x,t)}{\int dt S(x,t)}$$

Poynting-vector (could be Energy)

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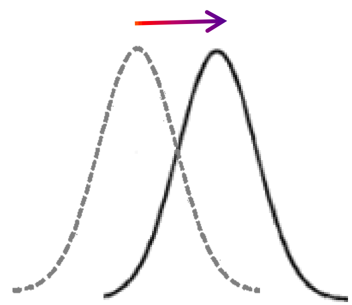

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good agreement ← values at detectors

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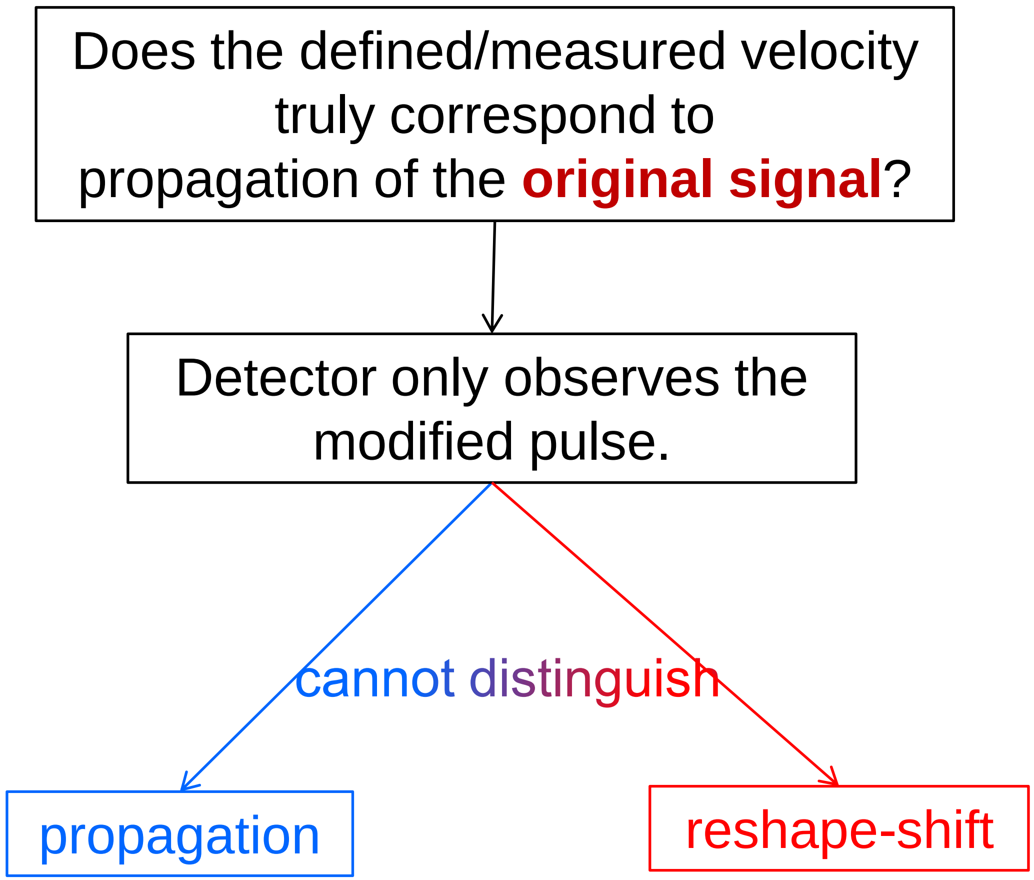
Is velocity true?



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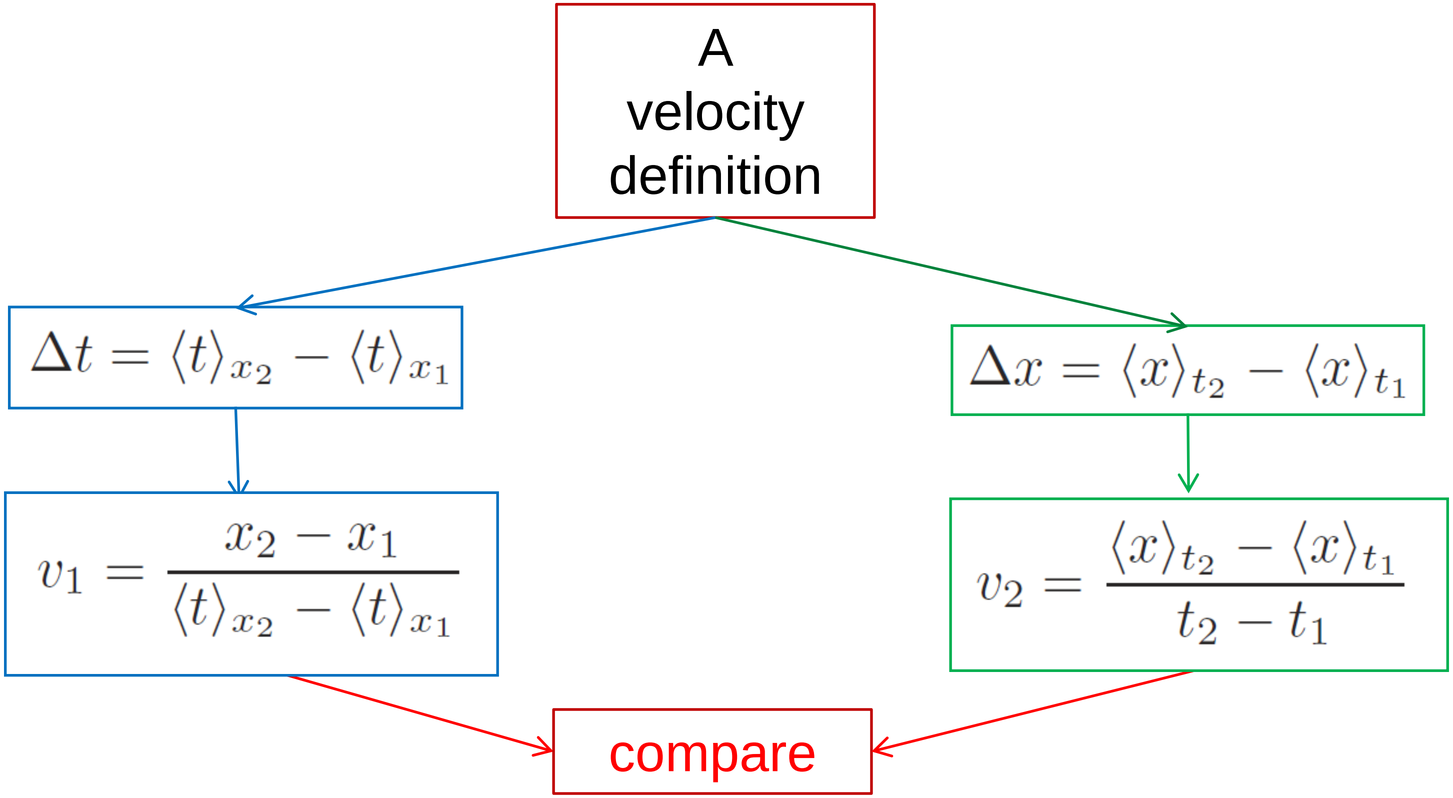
Method to test velocities

A velocity definition

$$\Delta t = \langle t \rangle_{x_2} - \langle t \rangle_{x_1}$$

$$v_1 = \frac{x_2 - x_1}{\langle t \rangle_{x_2} - \langle t \rangle_{x_1}}$$

Method to test velocities



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$$\Delta x = \langle x \rangle_{t_2} - \langle x \rangle_{t_1}$$

$$v_2 = \frac{\langle x \rangle_{t_2} - \langle x \rangle_{t_1}}{t_2 - t_1}$$

compare

if <x> or <t> movement is really due to flow

v_1 and v_2 must be very similar!

Fourier space to work within

$$\langle t \rangle_x = \frac{\int dt\, t\, S(x,t)}{\int dt S(x,t)}$$

can be calculated using real-ω expansion

$$\langle x \rangle_t = \frac{\int dx\, x\, S(x,t)}{\int dt S(x,t)}$$

can be calculated using real-k expansion

Fourier space to work within

$$\int dt\, t\, S(\Delta x, t) = \Delta x \int_{-\infty}^{+\infty} d\bar{\omega} \frac{dk}{d\omega} e^{-2k_I \Delta x} \left|D_1(\bar{\omega})\right|^2 n^*(\bar{\omega})$$
$$-i \int_{-\infty}^{+\infty} d\bar{\omega} e^{-2k_I \Delta x} \frac{dD_1}{d\omega} D_1^*(\bar{\omega}) n^*(\bar{\omega})$$

$$\langle t \rangle_x = \frac{\int dt\, t\, S(x,t)}{\int dt S(x,t)}$$

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$$\langle x \rangle_t = \frac{\int dx\, x\, S(x,t)}{\int dt S(x,t)}$$

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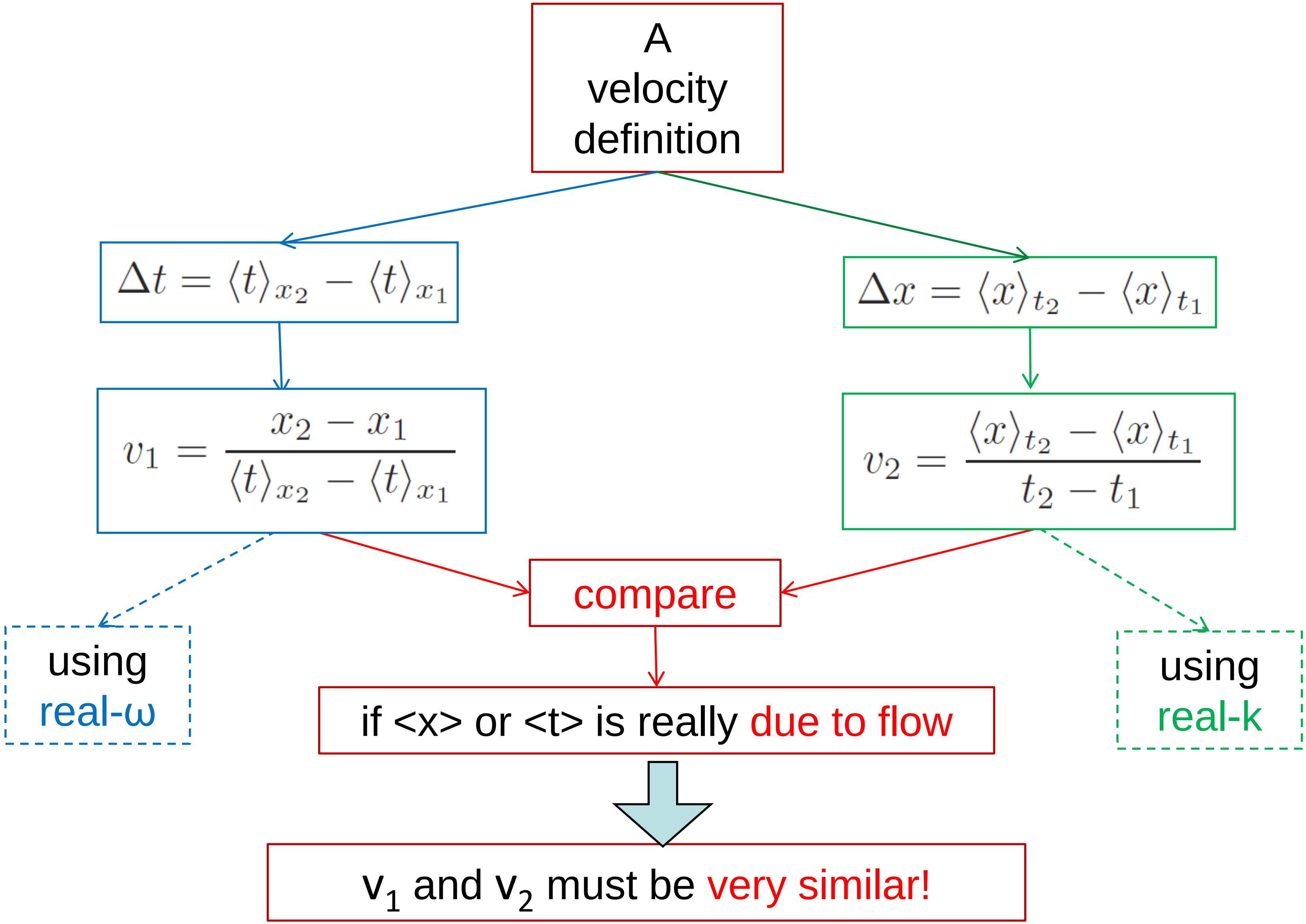
$$\int dx\, x\, S(x, \Delta t) = \Delta t \int_{-\infty}^{+\infty} d\bar{k} \frac{d\omega}{dk} e^{2\omega_I \Delta t} \left|D_2(\bar{k})\right|^2 n^*(\bar{k})$$
$$+i \int_{-\infty}^{+\infty} d\bar{k} e^{2\omega_I \Delta t} \frac{dD_2}{dk} D_2^*(\bar{k}) n^*(\bar{k})$$

Method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$\Delta x = \langle x \rangle_{t_2} - \langle x \rangle_{t_1}$
$v_1 = \frac{x_2 - x_1}{\langle t \rangle_{x_2} - \langle t \rangle_{x_1}}$
$v_2 = \frac{\langle x \rangle_{t_2} - \langle x \rangle_{t_1}}{t_2 - t_1}$
using real-ω
compare
using real-k
if <x> or <t> is really due to flow
v_1 and v_2 must be very similar!

in order to compare

$$\int dt\, t\, S(\Delta x, t) = \Delta x \int_{-\infty}^{+\infty} d\bar{\omega} \frac{dk}{d\omega} e^{-2k_I \Delta x} \left|D_1(\bar{\omega})\right|^2 n^*(\bar{\omega}) - i \int_{-\infty}^{+\infty} d\bar{\omega} e^{-2k_I \Delta x} \frac{dD_1}{d\omega} D_1^*(\bar{\omega}) n^*(\bar{\omega})$$

$$\langle t \rangle_x = \frac{\int dt\, t\, S(x,t)}{\int dt S(x,t)}$$

can be calculated using real-ω expansion

relate

$D_1(\omega) \leftrightarrow D_2(k)$

$$\langle x \rangle_t = \frac{\int dx\, x\, S(x,t)}{\int dt S(x,t)}$$

can be calculated using real-k expansion

$$\int dx\, x\, S(x, \Delta t) = \Delta t \int_{-\infty}^{+\infty} d\bar{k} \frac{d\omega}{dk} e^{2\omega_I \Delta t} \left|D_2(\bar{k})\right|^2 n^*(\bar{k}) + i \int_{-\infty}^{+\infty} d\bar{k} e^{2\omega_I \Delta t} \frac{dD_2}{dk} D_2^*(\bar{k}) n^*(\bar{k})$$

$D_1(\omega) \leftrightarrow D_2(k)$

a) real-ω

Left ($n = 1$):

$$\int_{-\infty}^{+\infty} d\bar{\omega} A_1(\bar{\omega}) e^{i(k(\bar{\omega})x-\bar{\omega}t)} + \int_{-\infty}^{+\infty} d\bar{\omega} B_1(\bar{\omega}) e^{i(k(\bar{\omega})x+\bar{\omega}t)}$$

Right ($n = n_R + i n_I$):

$$\int_{-\infty}^{+\infty} d\bar{\omega} D_1(\bar{\omega}) e^{i(k(\bar{\omega})x-\bar{\omega}t)}$$

$x = 0$

b) real-k

Left ($n = 1$):

$$\int_{-\infty}^{+\infty} d\bar{k} A_2(\bar{k}) e^{i(\bar{k}x-\omega(\bar{k})t)} + \int_{-\infty}^{+\infty} d\bar{k} B_2(\bar{k}) e^{i(\bar{k}x+\omega(\bar{k})t)}$$

Right ($n = n_R + i n_I$):

$$\int_{-\infty}^{+\infty} d\bar{k} D_2(\bar{k}) e^{i(\bar{k}x-\omega(\bar{k})t)}$$

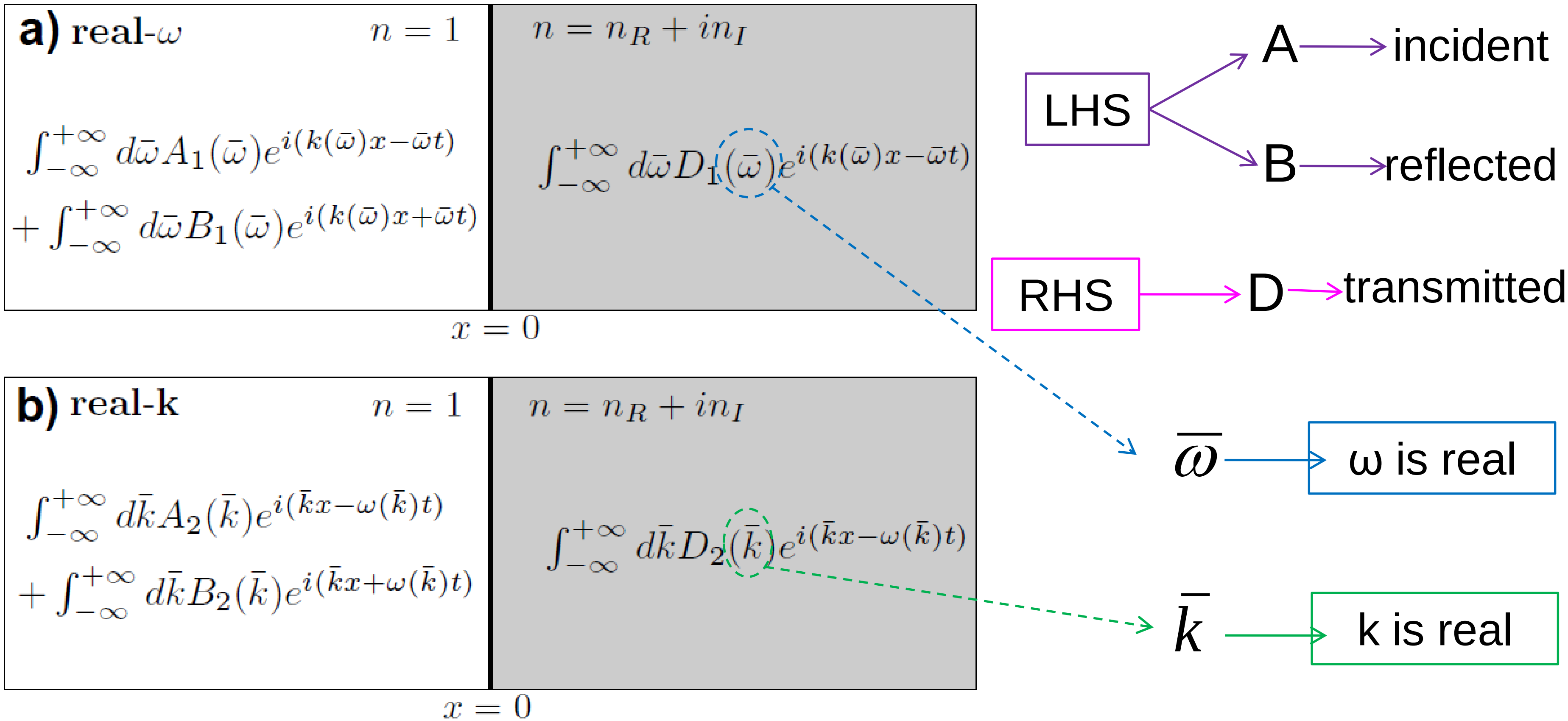
$x = 0$

LHS → A → incident

LHS → B → reflected

RHS → D → transmitted

$D_1(\omega) \leftrightarrow D_2(k)$



$D_1(\omega) \leftrightarrow D_2(k)$

a) real-ω $n = 1$ | $n = n_R + in_I$

$$\int_{-\infty}^{+\infty} d\bar{\omega} A_1(\bar{\omega}) e^{i(k(\bar{\omega})x - \bar{\omega}t)} + \int_{-\infty}^{+\infty} d\bar{\omega} B_1(\bar{\omega}) e^{i(k(\bar{\omega})x + \bar{\omega}t)}$$

$$\int_{-\infty}^{+\infty} d\bar{\omega} D_1(\bar{\omega}) e^{i(k(\bar{\omega})x - \bar{\omega}t)}$$

$x = 0$

b) real-k $n = 1$ | $n = n_R + in_I$

$$\int_{-\infty}^{+\infty} d\bar{k} A_2(\bar{k}) e^{i(\bar{k}x - \omega(\bar{k})t)} + \int_{-\infty}^{+\infty} d\bar{k} B_2(\bar{k}) e^{i(\bar{k}x + \omega(\bar{k})t)}$$

$$\int_{-\infty}^{+\infty} d\bar{k} D_2(\bar{k}) e^{i(\bar{k}x - \omega(\bar{k})t)}$$

$x = 0$

LHS → A → incident

LHS → B → reflected

RHS → D → transmitted

$\bar{\omega}$ → ω is real

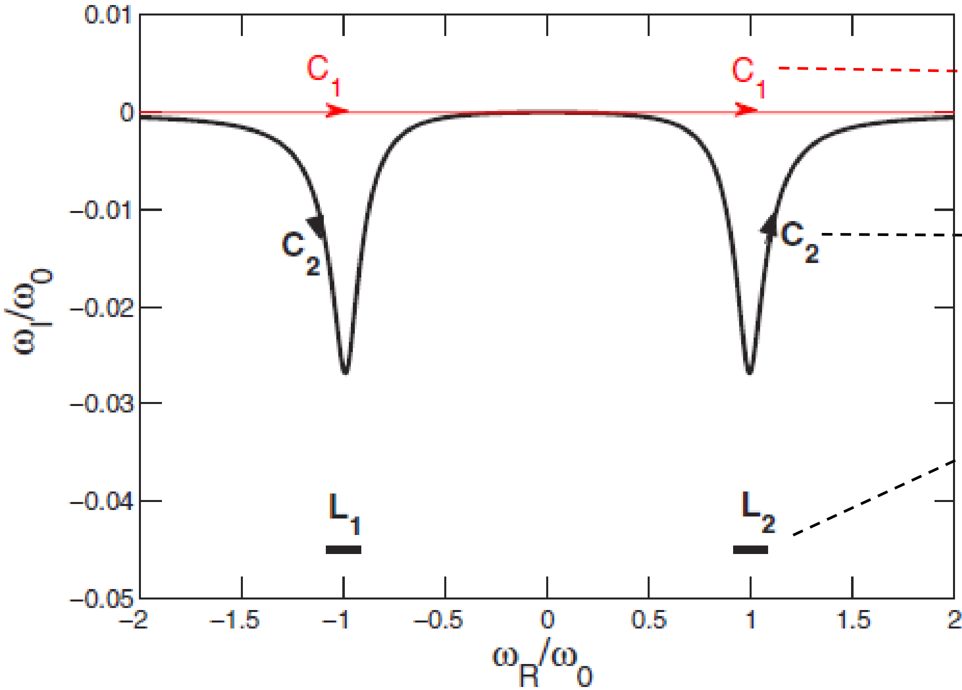
$\bar{k}$ → k is real

$$\int_{-\infty}^{+\infty} d\bar{\omega} D_1(\bar{\omega}) e^{-i\bar{\omega}t} = \int_{-\infty}^{+\infty} d\bar{k} D_2(\bar{k}) e^{-i\omega(\bar{k})t},$$

→ RHSs equal at x=0

$D_1(\omega) \leftrightarrow D_2(k)$

$$\int_{-\infty}^{+\infty} d\bar{\omega} D_1(\bar{\omega}) e^{-i\bar{\omega}t} = \int_{-\infty}^{+\infty} d\bar{k} D_2(\bar{k}) e^{-i\omega(\bar{k})t},$$



$\bar{\omega}$ → ω is real

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branch-cuts

$D_1(\omega) \leftrightarrow D_2(k)$

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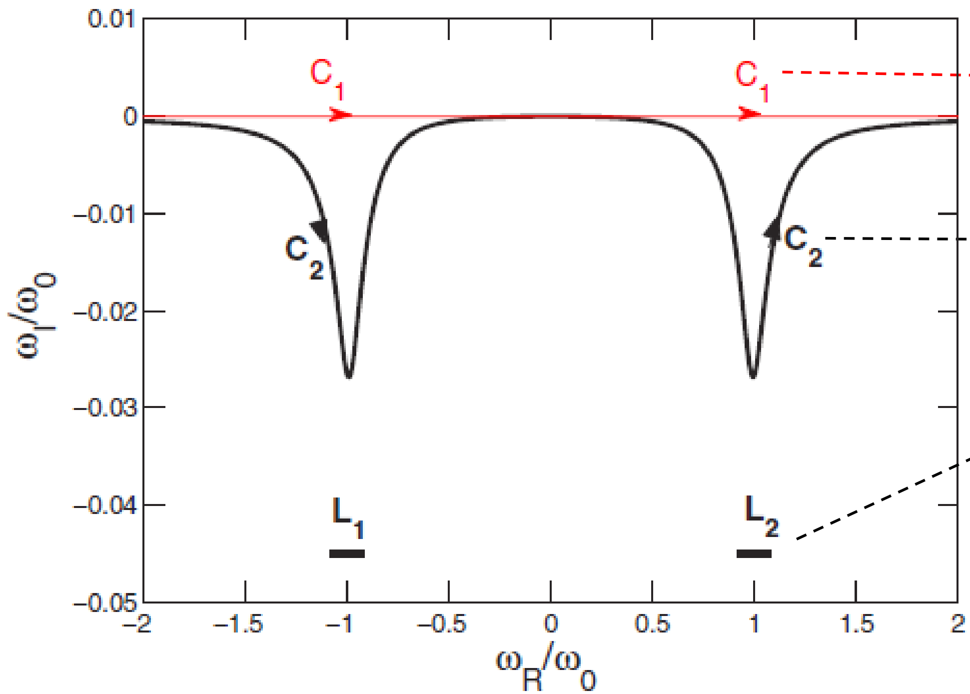


$\bar{\omega}$ → ω is real

$\bar{k}$ → k is real

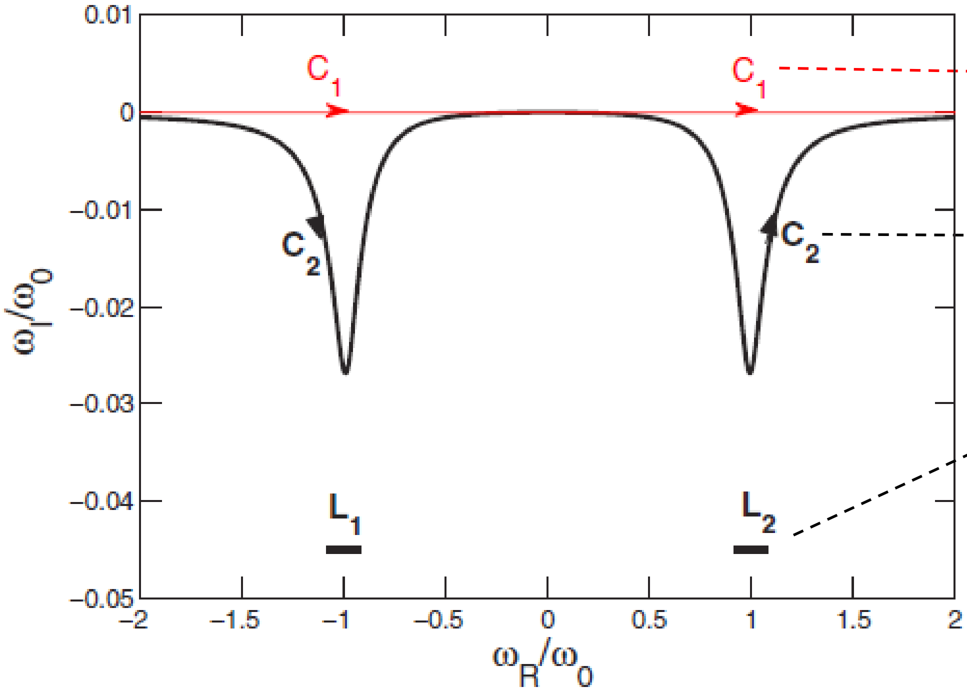
→ branch-cuts

n(ω) → no pole / no branch-cut → between C₁ and C₂

$$D_1(\omega) = \frac{2A(\omega)}{1 + n(\omega)}$$ → no pole / no branch-cut → between C_1 and C_2

$D_1(\omega) \leftrightarrow D_2(k)$

$$\int_{-\infty}^{+\infty} d\bar{\omega} D_1(\bar{\omega}) e^{-i\bar{\omega}t} = \int_{-\infty}^{+\infty} d\bar{k} D_2(\bar{k}) e^{-i\omega(\bar{k})t},$$



$\bar{\omega}$ → ω is real

$\bar{k}$ → k is real

branch-cuts

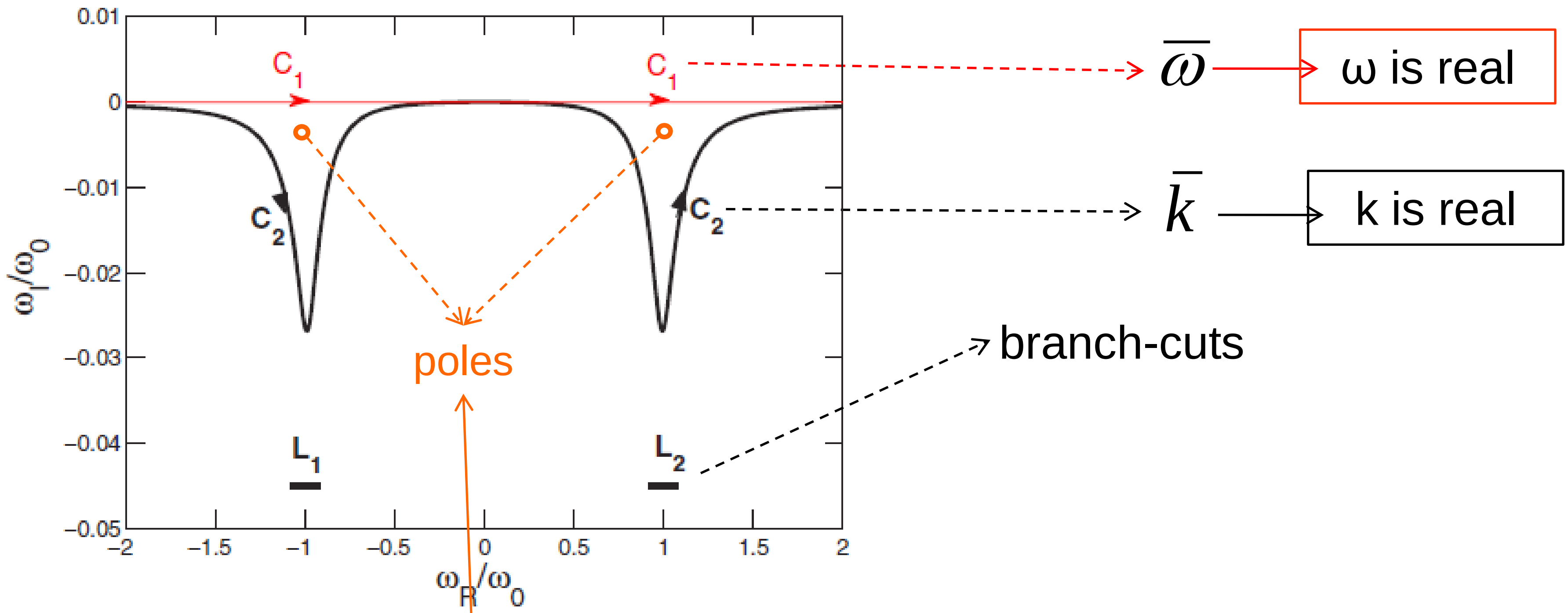
n(ω) → no pole, no branch-cut → between C_1 and C_2

$$D_2(\bar{k}) = \frac{d\omega}{dk}(\bar{k}) D_1(\omega(\bar{k})),$$

$$D_1(\omega) = \frac{2A(\omega)}{1+n(\omega)}$$

→ no pole, no branch-cut → between C_1 and C_2

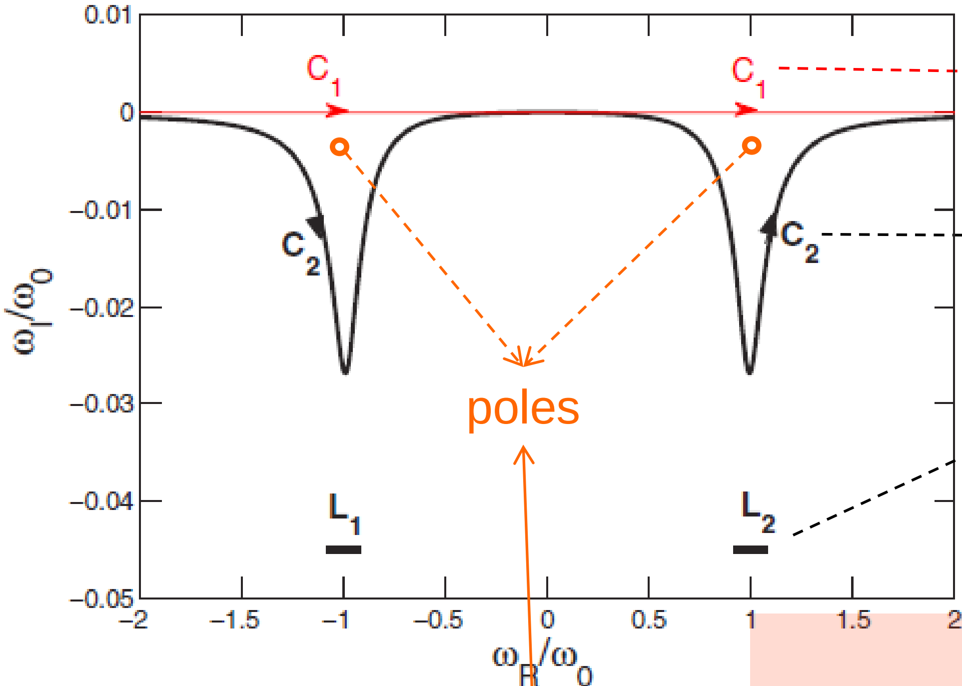
$D_1(\omega) \leftrightarrow D_2(k)$ (if poles)



$$D_1(\omega) = \frac{2A(\omega)}{1 + n(\omega)}$$

has poles

$D_1(\omega) \leftrightarrow D_2(k)$ (if poles)



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$$D_1(\omega) = \frac{2A(\omega)}{1+n(\omega)}$$

has poles

$$D_1(\bar\omega) = \left(\frac{dk}{d\omega}\right)_{\bar\omega} D_2\left(\frac{n(\bar\omega)\bar\omega}{c}\right)$$

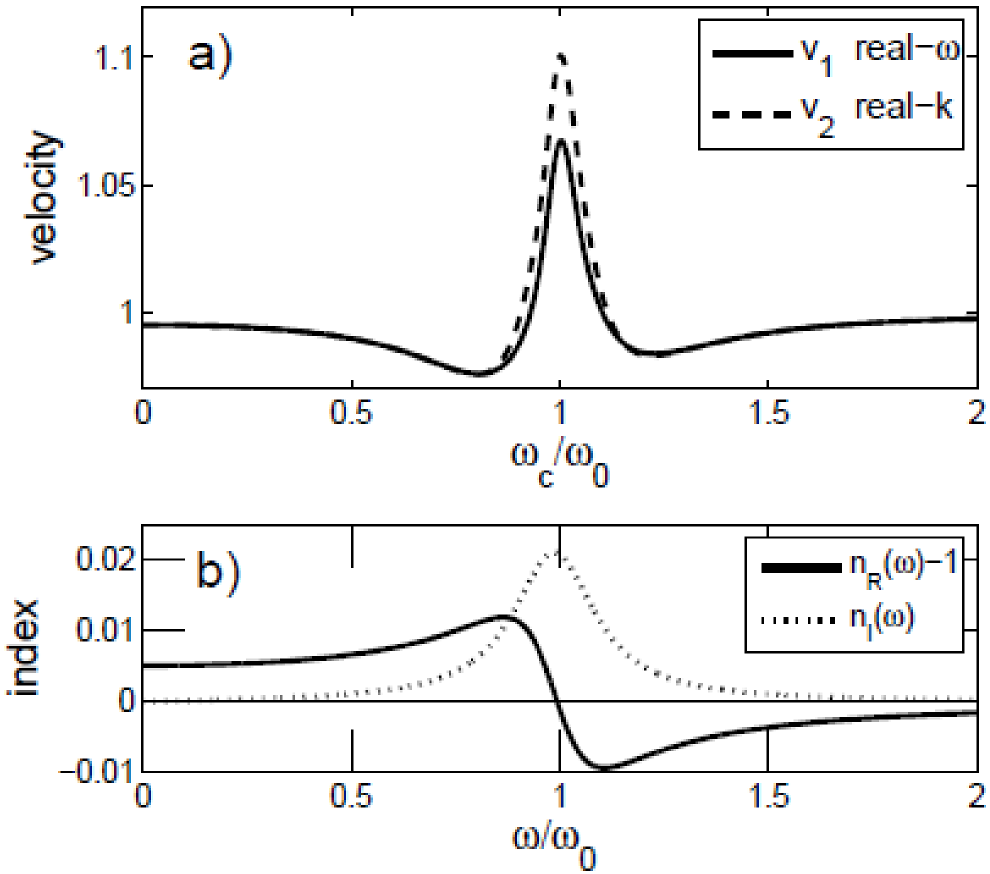
$$+\ 2\pi i \frac{F(\omega_{1R} - i\omega_{1I})}{2\omega_{1R}} \frac{e^{i(\omega_{1R}-\bar\omega)T} e^{\omega_{1I}T}}{2\pi i(\omega_1-\bar\omega)}$$

$$+\ 2\pi i \frac{F(-\omega_{2R} - i\omega_{2I})}{-2\omega_{2R}} \frac{e^{i(-\omega_{2R}-\bar\omega)T} e^{\omega_{2I}T}}{2\pi i(\omega_2-\bar\omega)}$$

$$-\ \lim_{T\to\infty}\sum_{1,2}\oint_{C_{1,2}} d\omega \left(\frac{dk}{d\omega}\right) D_2\left(\frac{n(\omega)\omega}{c}\right)\frac{e^{-i(\omega-\bar\omega)T}}{2\pi i(\omega-\bar\omega)}$$

Comparison of v_1 and v_2

Gaussian wave-packet → $U(0,t) = e^{-t^2/\tau^2}\cos(\omega_c t)$



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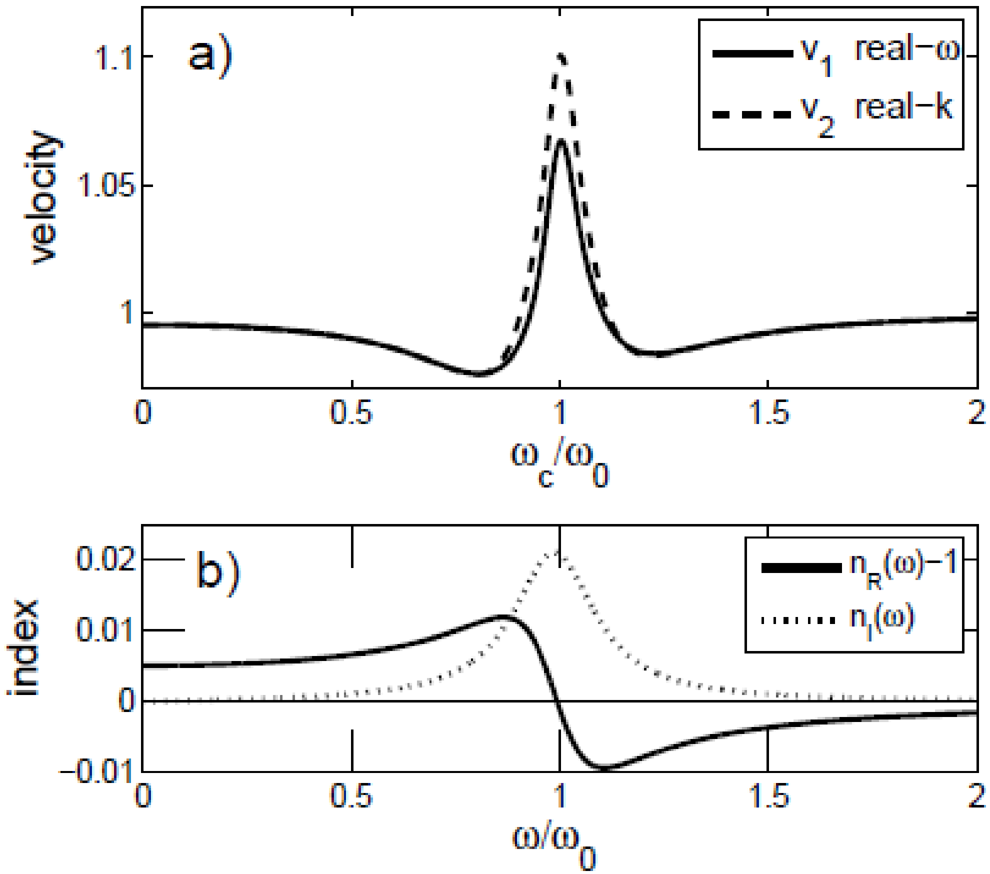
Luminal regime → $v_1 \cong v_2$



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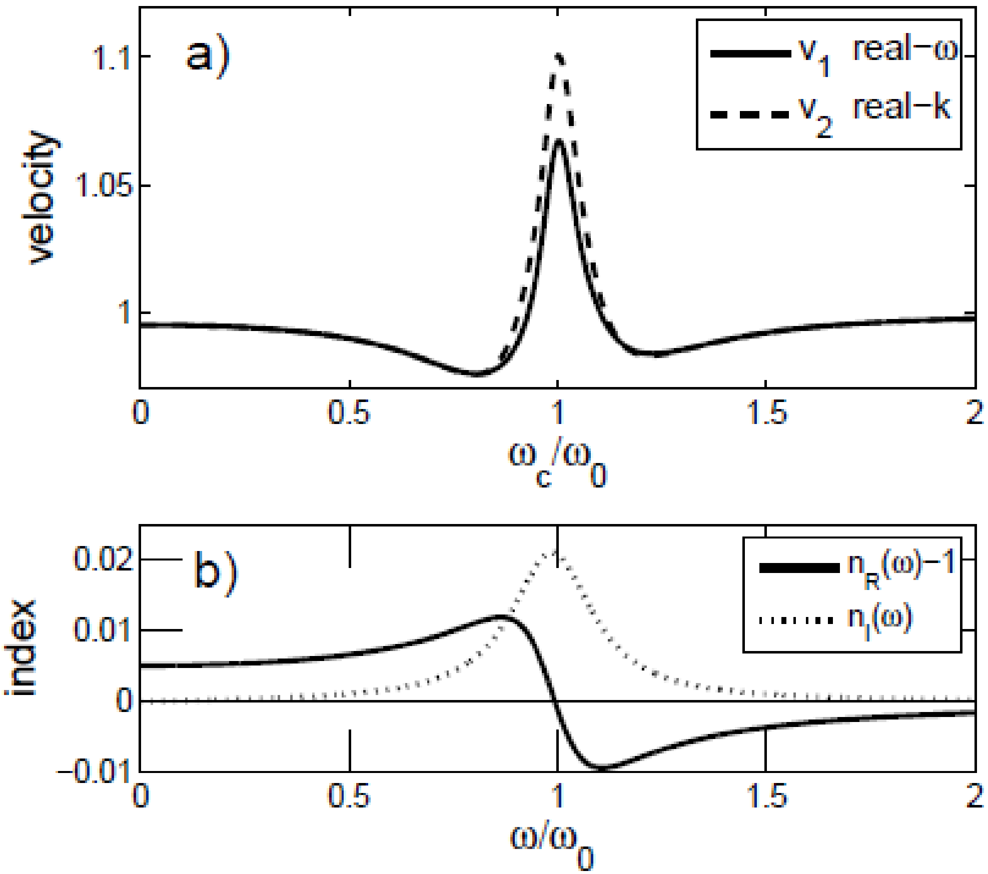
at resonance ($\omega_c \sim \omega_0$)

both v_1 and v_2 superluminal



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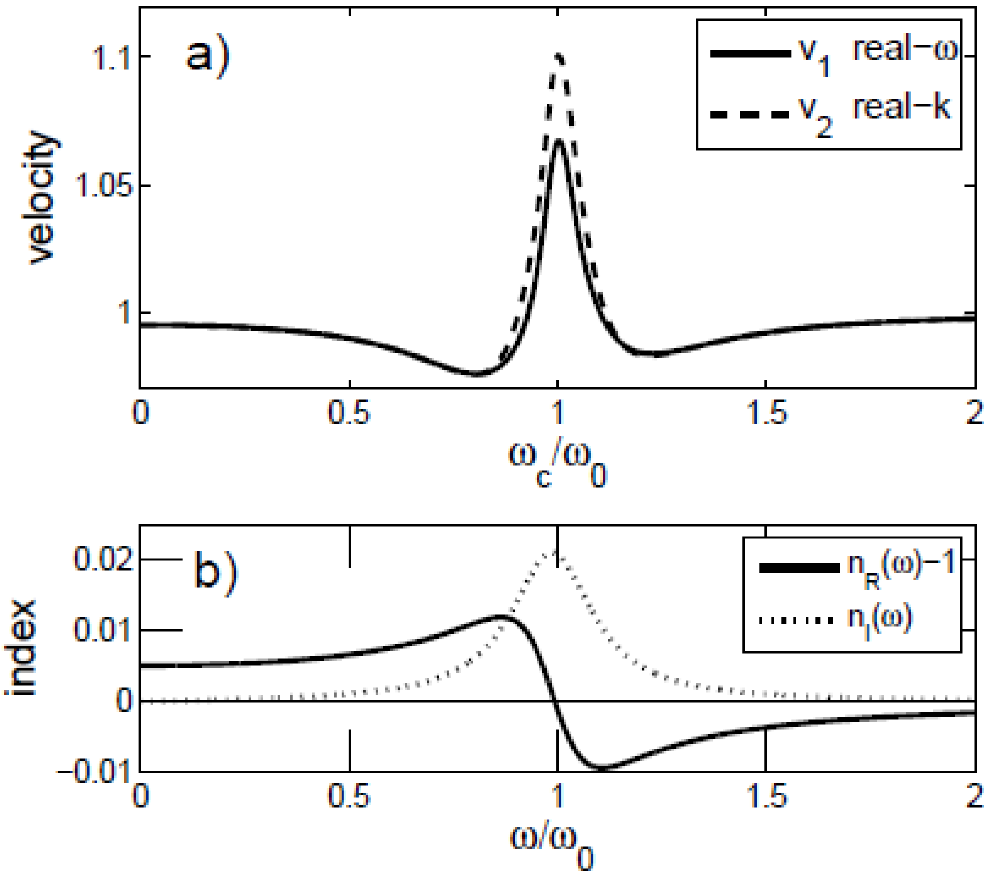
both v_1 and v_2 superluminal

however v_1 , v_2 differs



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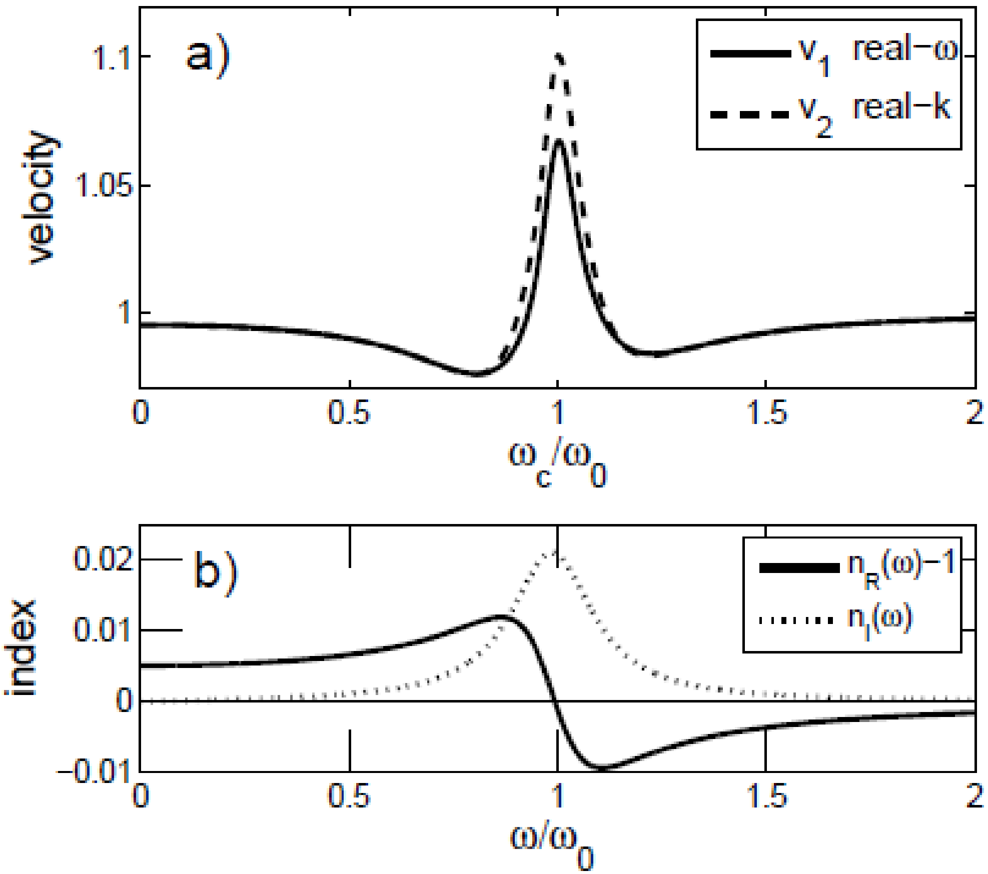

at resonance ($\omega_c \sim \omega_0$)

both v_1 and v_2 superluminal

however

v_1 , v_2 differs

velocity definition is inconsistent

not reliable

not correspond to a real flow

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Experiment again

Nanda *et al.* [3]

showed

$$\langle t \rangle_x = \frac{\int dt\, t\, S(x,t)}{\int dt S(x,t)}$$

corresponds to

detection time

[3] Lipsa Nanda, Aakash Basu, and S. A. Ramakrishna, Phys. Rev. E 74, 036601 (2006).

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showed

$$\langle t \rangle_x = \frac{\int dt\, t\, S(x,t)}{\int dt S(x,t)}$$

corresponds to detection time

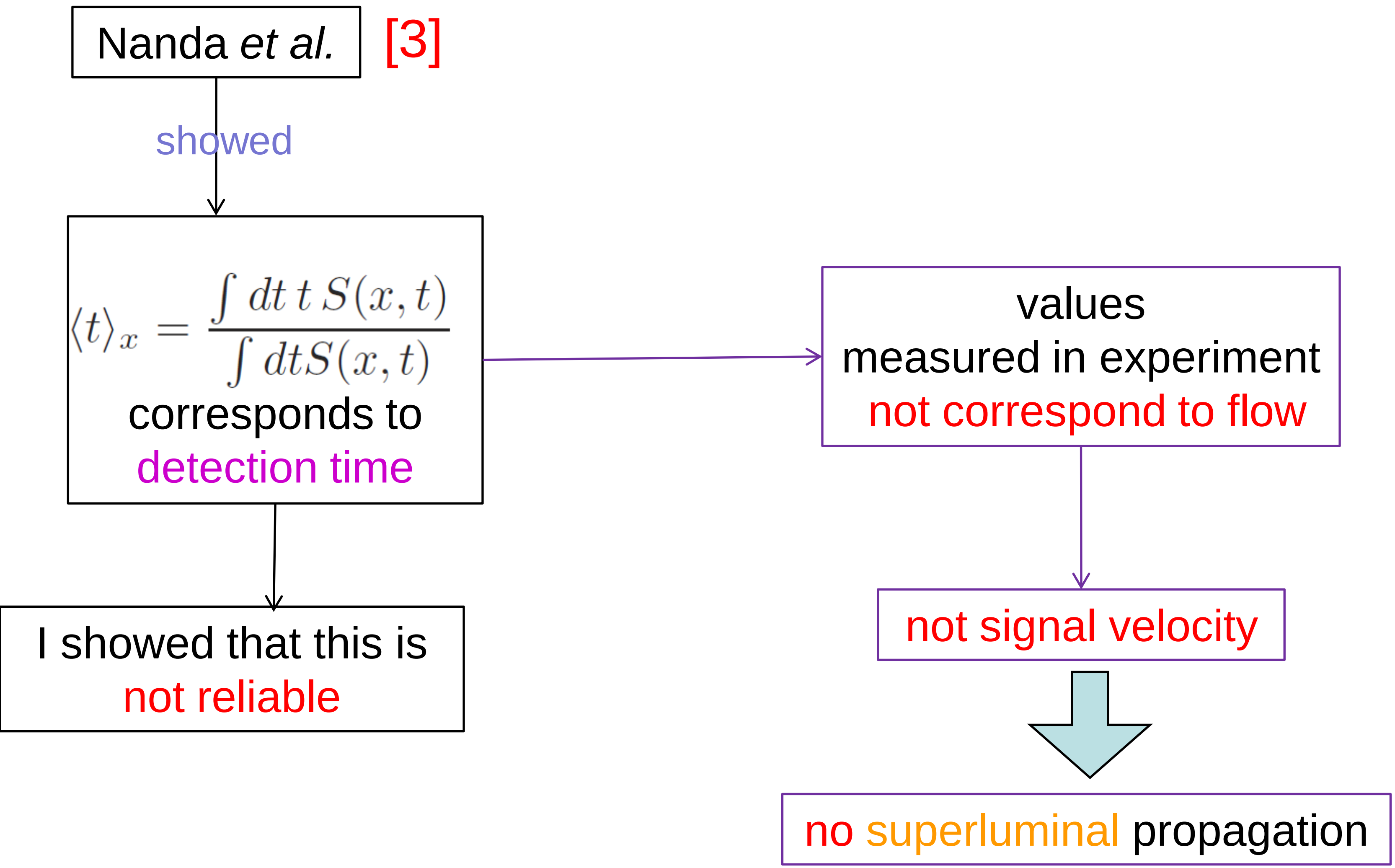
I showed that this is not reliable

values measured in experiment not correspond to flow

not signal velocity

[3] Lipsa Nanda, Aakash Basu, and S. A. Ramakrishna, Phys. Rev. E 74, 036601 (2006).

Experiment again



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Summary

- Cannot distinguish between propagation and reshaping.
- Signal velocity and Pulse-peak velocity differ.
- Introduced a method to check if a velocity corresponds a physical flow?
- Detectors measure pulse-peak velocity.
- Observed is not superluminal propagation; it's reshaping.

Acknowledgement

- Special thanks to Victor Kozlov for illuminating discussions.
- I thank Gürsoy Akgüç for intensive help in the manuscript.

TÜBİTAK

TUBİTAK-KARİYER No: 112T927

TÜBİTAK-1001 No: 110T876