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Research Article

Comparative Controlling of the Lorenz Chaotic System Using the SMC and APP Methods

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The Lorenz chaotic system is based on a nonlinear behavior and this causes the system to be unstable. Therefore, two different controller models were developed and named as the adaptive pole placement and sliding mode control (SMC) methods for the establishment of continuous time nonlinear Lorenz chaotic system. In order to achieve this, an improved controller structure was developed first theoretically for both the controller methods and then tested practically using the numerical samples. During the establishment of adaptive pole placement method for the Lorenz chaotic system, various stages were applied. The nonlinear chaotic system was also linearized by means of Taylor Series expansion processes. In addition, the feedback matrix of the adaptive pole placement method was determined using linear Jacobian matrix. The chaotic system reached an equilibrium point by using both the SMC and adaptive pole placement methods; however the simulation results of the SMC had better success than adaptive pole placement control technique.

1. Introduction

Several studies have been conducted to analyze and control the chaotic structure since it has been found [1–4]. These studies are mainly focused on obtaining different chaotic structures, development process in the fields of applications on chaotic structures, and controlling the chaos with different procedures [5].

Lorenz, Chua, Rössler, Rikitake, Rucklidge, Chen, Lü, and Genesio developed the most important chaotic systems. These systems were applied to many control methods, such as OGY [6], nonlinear feedback control [7], delay feedback control [8], sliding mode control [9], switching control [10], fuzzy sliding mode control [11], and adaptive backstepping control [12].

Chaos is illustrated by nonlinear behaviors; therefore SMC technique can be used to control of the chaotic systems. SMC technique has been used by many different system controls, such as robotic, mechatronic, machine driver, chemical processes, wind turbine systems, and DC/DC converter [13].

The structure of the SMC must be known in order to understand the applications of the SMC technology. SMC has two different stages known as reaching and sliding phase. Firstly, a proper slip surface is the choice for the sliding mode technique. Switching technique forms the basic structure of SMC. In contrast to the switching ratio ideal, SMC is limited by physical reasons. The most important problem in these systems is chattering which is caused by fast switching. Switching functions such as relay, sigmoid, saturation, hysteresis-saturation, and hyperbolic functions could be used to reduce the chattering level [13, 14].

Abundant literature is available regarding the linearization considered as an important method for controlling of nonlinear systems. One of the most important studies about time-invariant systems linearization was conducted by Khalil [15]. Another study also addressed the problem of approximate linearization of a nonlinear control system [16].

The adaptive pole placement-based controller technique is another important method used in this work. A zero equilibrium point was selected for APPM. The system which is nonlinear is linearized around this point, and required feedback vector is obtained from Taylor's series.

The applications of the pole placement-based controller techniques were examined in literature. Some of important applications related to this topic are as follows. Chilali and Gahinet have presented a novel LMI characterization for general convex subregions of the complex plane and proved its practicality for H∞ synthesis with closed-loop pole clustering constraints [17]. Another study presented a PPM using both the improved Jacobian and the corresponding system transfer function matrices [18].

On the other hand, the control of chaotic systems with nonlinear behavior has become an important engineering problem recently. At the same time, chaotic systems have been widely used to explain the events and systems involving nonlinear behaviors such as Lorenz's description of weather events. In addition, chaotic systems provide a very important infrastructure for encrypting and sending electronic data that are widely used in communication systems. The aim of this study is to present a different approach to control the chaotic system of Lorenz, which is one of the basic studies for chaotic systems. The performances of two different control methods to accomplish this aim have been compared. The results of such comparisons would be a good example of the application of new control techniques on chaotic systems. It will also give a different approach to the chaotic structures used industrially.

This paper has been organized as follows. First, a brief definition of a Lorenz chaos system is given in Section 2. Then, design of sliding mode controller is given in Section 3. Afterward, numerical simulations for chaos control by way of sliding mode control and adaptive pole placement methods are given. Finally, conclusion is given in Section 5.

2. The Modeling of the Lorenz Chaotic System

The Lorenz system with a nonlinear structure is described by (1) given below. While positive constant parameters are a, b, and c, state variables are x, y, and z. The typical literature parameter values of the a, b, and c constants are a=10, b=8/3, and c=28, respectively.

$$\begin{aligned} \dot{x} &= a.(-x+y) \\ \dot{y} &= (c-z).x - y \\ \dot{z} &= x.y - b.z \end{aligned} \tag{1}$$

The Lorenz chaotic system of the xy, xz, yz, and xyz phase portraits were obtained by using a MATLAB/Simulink program as indicated in Figure 1, when $x_0 = 0.001$, $y_0 = 0.001$, and $z_0 = 0$. The initial values of the chaotic system were selected as values close or similar to the values in the literature. The choice of initial values is very important because of the change in all dynamic behavior.

3. SMC Design for Lorenz Chaotic System

SMC may be practiced for the Lorenz chaotic system. Stability of the Lorenz system was improved using only one controller. The system can be presented with

$$\begin{aligned} \dot{x} &= a.(y-x) \\ \dot{y} &= x.(c-z) - y + u \\ \dot{z} &= x.y - b.z \end{aligned} \tag{2}$$

where u is the control input.

After choosing of a sliding surface like (3), equations indicated below might be established:

$$s = \dot{e} + \lambda.e \tag{3}$$

$$\dot{s} = \ddot{e} + \lambda.\dot{e} \tag{4}$$

The trajectory error state could be selected like $e = y_r - y$, where y_r is constant, so $\dot{y}_r = \ddot{y}_r = 0$. $\dot{y}_r$ and $\ddot{y}_r$ are obtained as $\dot{e} = \dot{y}_r - \dot{y} = 0 - \dot{y} = -\dot{y}$ and $\ddot{e} = \ddot{y}_r - \ddot{y} = 0 - \ddot{y} = -\ddot{y}$.

$$\dot{s} = -\rho.\,\text{sign}(s) \tag{5}$$

After a proportional reachability rule as (5) is selected, equations below may be inscribed.

$$\begin{aligned} \dot{s} = \ddot{e} + \lambda.\dot{e} &= \ddot{y}_r - \ddot{y} + \lambda.(\dot{y}_r - \dot{y}) \\ &= -\rho.\,\text{sign}(s) \\ -\ddot{y} - \lambda.\dot{y} &= -\rho.\,\text{sign}(s) \\ \dot{y}_{new} &= \dot{y} + u \\ -\ddot{y} - \lambda.\dot{y} - \lambda.u &= -\rho.\,\text{sign}(s) \end{aligned} \tag{6}$$

The control input is provided like

$$u = -\frac{\ddot{y}}{\lambda} - \dot{y} + \frac{\rho.\,\text{sign}(s)}{\lambda} \tag{7}$$

The stability analysis is significant for evaluating the design of nonlinear controller. Therefore, Lyapunov stability analysis is selected and applied to SMC stability analysis. The stability is guaranteed, after the derivation of the Lyapunov function is negative definite [17].

$$\begin{aligned} &\dot{v}(t) = s.\dot{s} \le 0, s(t) \neq 0 \\ &s.\dot{s} = s.(-\rho.\,\text{sign}(s)) \le |s|.(-\rho.\,\text{sign}(s)) < 0 \end{aligned} \tag{8}$$

According to the Lyapunov 2^{nd} method, n to be constant, if $\lim_{t\longrightarrow\infty} y(t) = n$ is provided, then the systems is considered to be stable [19].

4. Adaptive Pole Placement Method Definitions

In the modern control theory and design, pole placement control techniques have been widely used. Firstly, the nonlinear Lorenz chaotic system is linearized by means of using

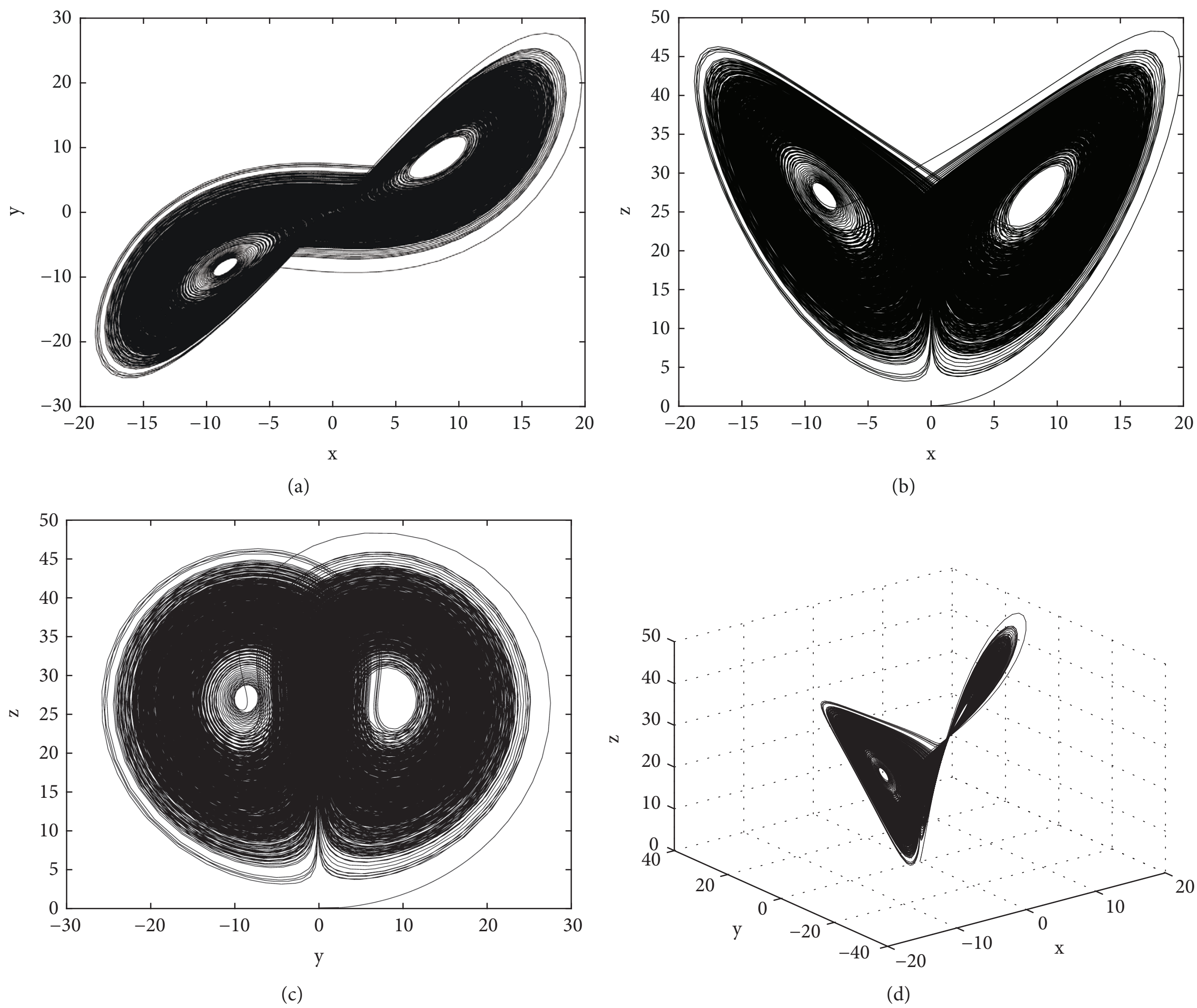

Figure 1: Phase portrait of the Lorenz systems in (a) xy, (b) xz, (c) yz, and (d) xyz.

Jacobian matrix, and then the feedback vector $\mathbf{K}[\mathbf{k_1}\ \mathbf{k_2}\ \mathbf{k_3}]$ is determined by using embedded programming including linear matrix. The feedback linearized system may be defined:

$$\mathbf{u}(\mathbf{t}) = -\mathbf{Kx}(\mathbf{t}) \tag{9}$$

Modifying the system equation too:

$$\dot{\mathbf{x}}(\mathbf{t}) = (\mathbf{A} - \mathbf{BK})\,\mathbf{x}(\mathbf{t}) + \mathbf{B}r(t) \tag{10}$$

The desired eigenvalues are to be at

$$|\mathbf{sI} - \mathbf{A} + \mathbf{BK}| = (s-\gamma_1)(s-\gamma_2)\dots(s-\gamma_n) \tag{11}$$

5. Numerical Simulation

The control input signal is obtained by using (9), where $\ddot{y} = -1$, $\dot{y} = 28x - x.z + y$, and SMC gains have been selected as $\lambda = 3$, $\rho = 0.01$ with the initial conditions $x_0 = 0.001$, $y_0 = 0.001$, and $z_0 = 0$. The controllers are activated at t= 40 seconds in all simulations. The system was linearized by means of Taylor's series and then the Jacobian matrix was obtained by using the first terms of linear elements. A function is expressed in a great ratio by the first terms of the expansion. Therefore, higher degree terms can be ignored.

$$u = -\frac{1}{\lambda} - 28x + xz + y + \frac{\rho.\,\text{sign}(s)}{\lambda}$$
$$u = -\frac{1}{3} - 28x + xz + y + \frac{0.01.\,\text{sign}(s)}{3} \tag{12}$$

$$f_1 = \dot{x} = a.(y - x)$$
$$f_2 = \dot{y} = x.(c - z) - y \tag{13}$$
$$f_3 = \dot{z} = x.y - b.z$$

$$f(x) = f(\overline{x}) = \left.\frac{df}{dx}\right|_{x=\overline{x}} (x - \overline{x}) + \text{Higher degree terms} \tag{14}$$

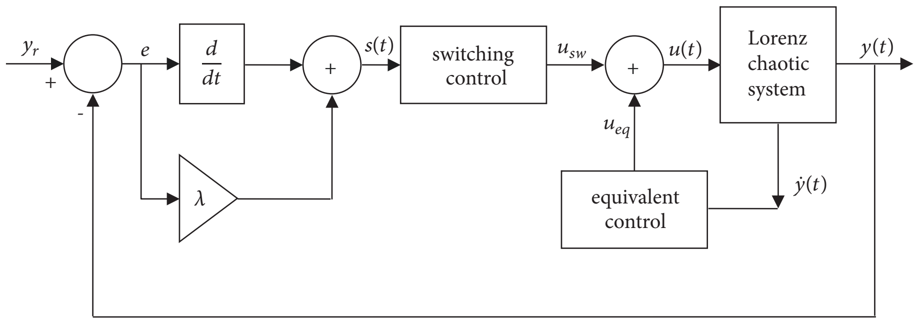

Figure 2: The SMC model for the Lorenz chaotic system.

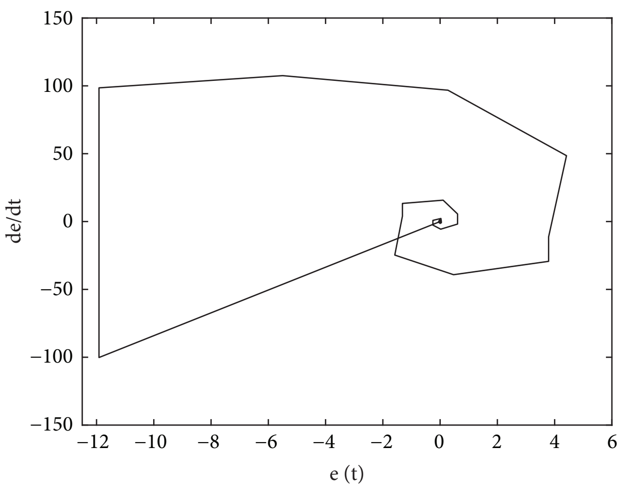

Figure 3: The phase planes e(t) and ė(t) in the Lorenz chaotic system.

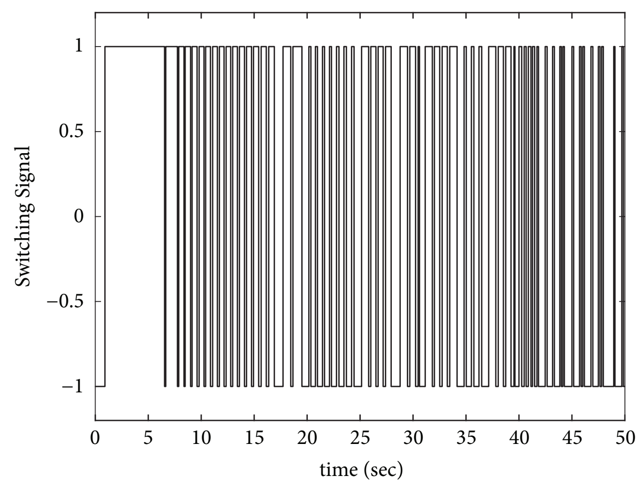

Figure 4: Switching signal for sliding mode control.

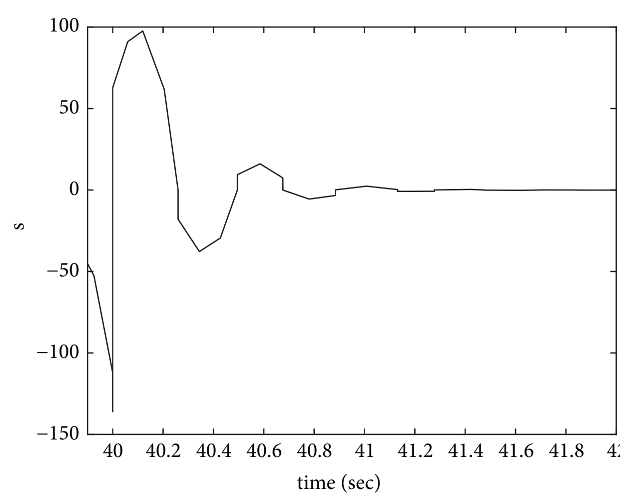

Figure 5: The time variation of the sliding surface.

The eigenvalues are calculated by means of solving the characteristic equation:

$$\mathrm{A} = |\lambda \mathrm{I} - \mathrm{J}| = \begin{vmatrix} \lambda - (-10) & -10 & 0 \\ z - 28 & 1 + \lambda & x \\ -y & -x & \lambda + \left(\frac{8}{3}\right) \end{vmatrix} = 0 \qquad (15)$$

Initial eigenvalues of $\lambda_1 = 22.8277$, $\lambda_2 = -11.8277$, $\lambda_3 = 2.6667$ for $x_0 = 0.001$, $y_0 = 0.001$, and $z_0 = 0$.

A suggested control model based on the equivalent control $u_{eq}(t)$ and switching control $u_{sw}(t)$ is indicated in Figure 2 and (16), (17), and (18), where ρ_{sw} is a positive constant.

$$u(t) = u_{eq}(t) + u_{sw}(t) \qquad (16)$$

$$u_{sw}(t) = -\rho_{sw} \operatorname{sign}(s(t)) \qquad (17)$$

$$u_{eq}(t) = u(t) - u_{sw}(t) \qquad (18)$$

In Figure 3, the phase system controlling sliding mode reached the third region from second region at fourth second. Then, the error of system reached the equilibrium point (zero point) after sliding phase.

High frequency control signal was applied to input of chaos system as shown in Figures 4, 5, and 6.

After the controller enters the system at 40th second as shown in Figure 7, the state variables reached an equilibrium point $E_0(0, 0, 0)$. This phenomenon is clearer in Figure 8 of the phase portrait.

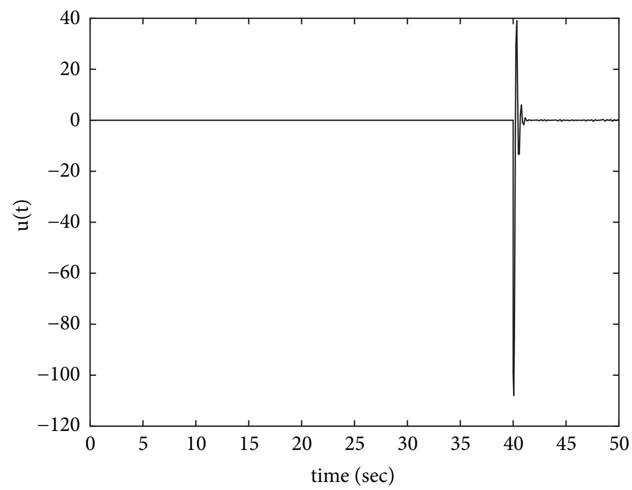

Figure 6: The control signals for sliding mode.

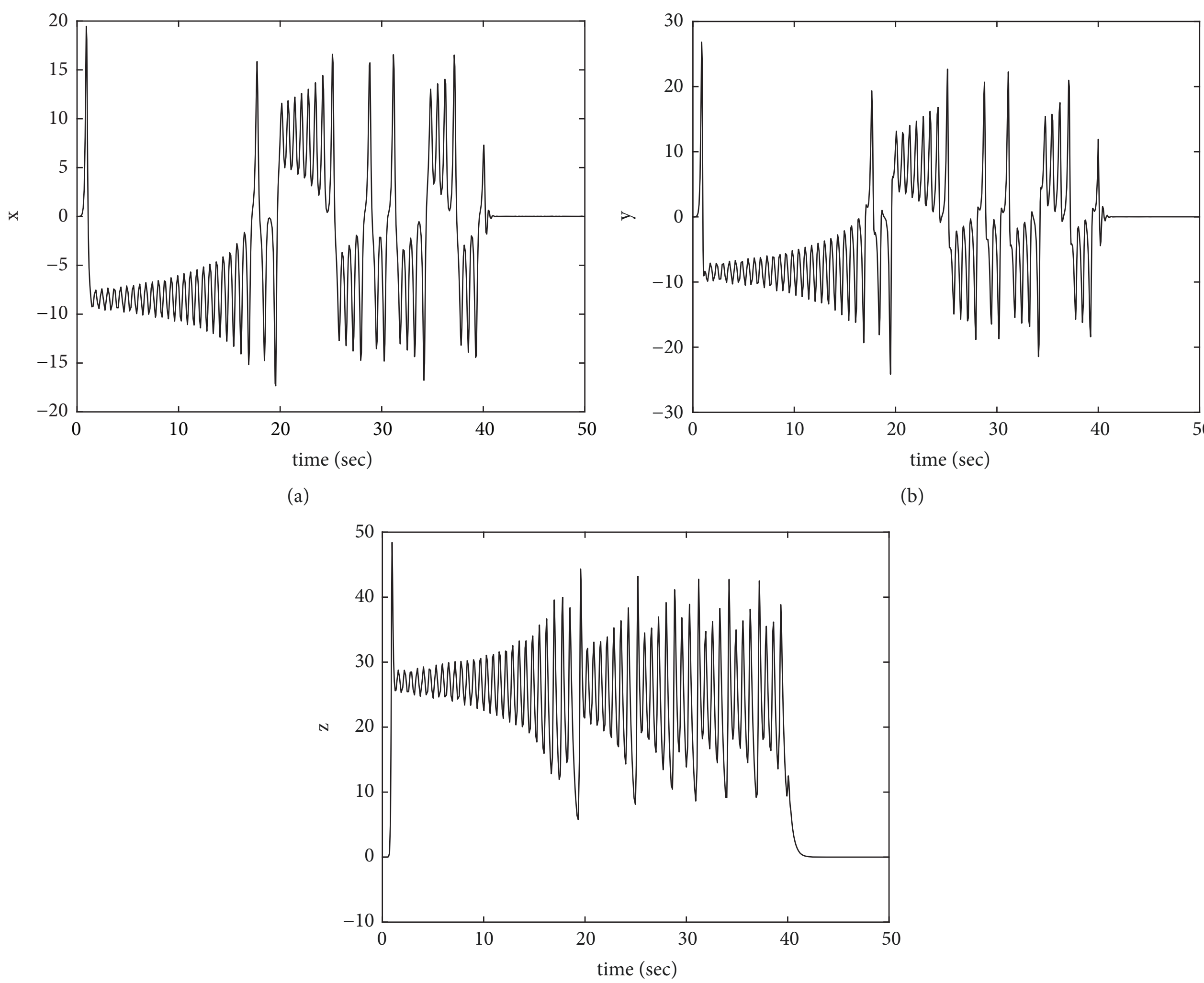

Figure 7: The time response of (a) x, (b) y, and (c) z state variables with the SMC activated at t = 40 seconds.

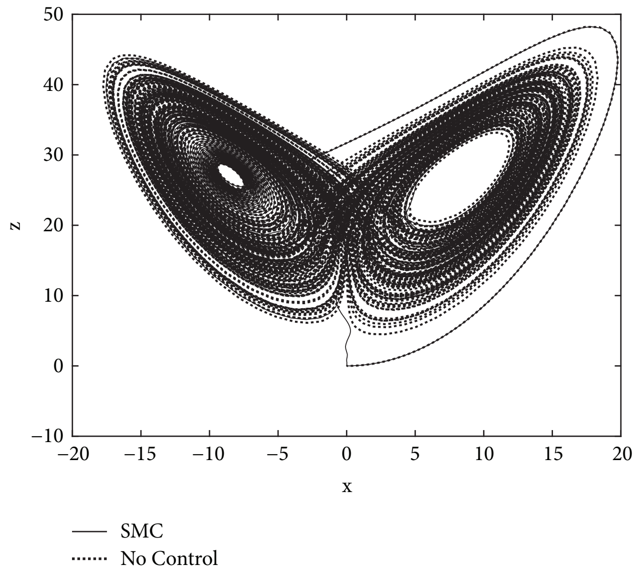

FIGURE 8: xz portraits with and without sliding mode control.

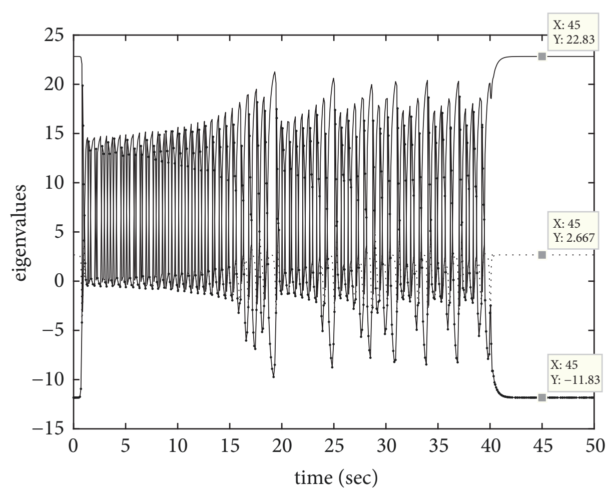

FIGURE 9: Eigenvalues changes with controller active at t=40 seconds.

In Figure 9, the eigenvalues of $E_0(0,0,0)$ are 22.83, 2.667, and -11.83, which are given respectively. The same values can also be reached at the equilibrium point due to the existence of the controller at 40th second. According to Lyapunov stability theorem, the criterion of $s.\dot{s} \leq 0$ was applied at fourth second and the system reached the stability. Initial eigenvalues of $\gamma_1 = 22.8277$, $\gamma_2 = -11.8277$, and $\gamma_3 = 2.6667$ were chosen for the adaptive pole placement system design. Using (14) adaptive pole placement method was improved as shown in Figure 10.

In Figure 11, the state variables reached the equilibrium point $E_0(0,0,0)$ quickly with an error. These conditions were also confirmed in both Figures 12 and 13 with attained eigenvalues, 22.81, 2.667, and -11.81, respectively. Also, $\mathbf{k_1}$, $\mathbf{k_2}$, and $\mathbf{k_3}$ are adaptive controller gains as shown in Figure 14. While the $\mathbf{k_2}$ feedback vector was fixed, the feedback vectors $\mathbf{k_1}$ and $\mathbf{k_3}$ varied intensively with the existence of the controller especially at 40^{th} second.

6. Conclusion

In this study, the controllers were developed by way of the SMC and APP methods for continuous time nonlinear Lorenz chaotic system. The simulation results of the SMC technique are more influential than APP technique. While the SMC method reached an equilibrium point, adaptive pole placement method reached an equilibrium point with greater error. According to SMC, the signal responses given by APPM are quite noisy and the steady-state errors have quite high values. Moreover, the results of the study showed that the system behaviors based on Lyapunov stability analysis were unstable and nonlinear at 0th to 40th second. The SMC, which is widely preferred in the literature, has performed very well in the control of the Lorenz chaotic system. Another recommended control method is APPM. This method can be used to control the Lorenz chaotic system but has a lower level of performance.

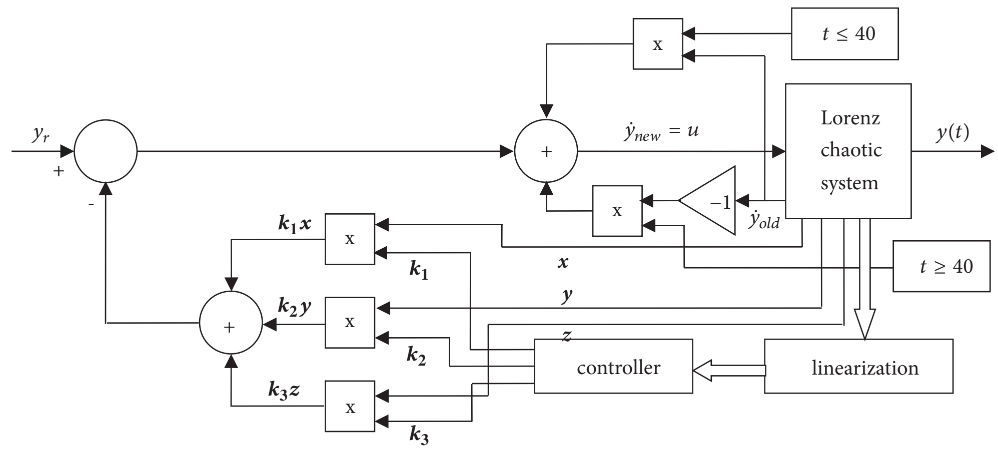

Figure 10: Proposed adaptive pole placement control method.

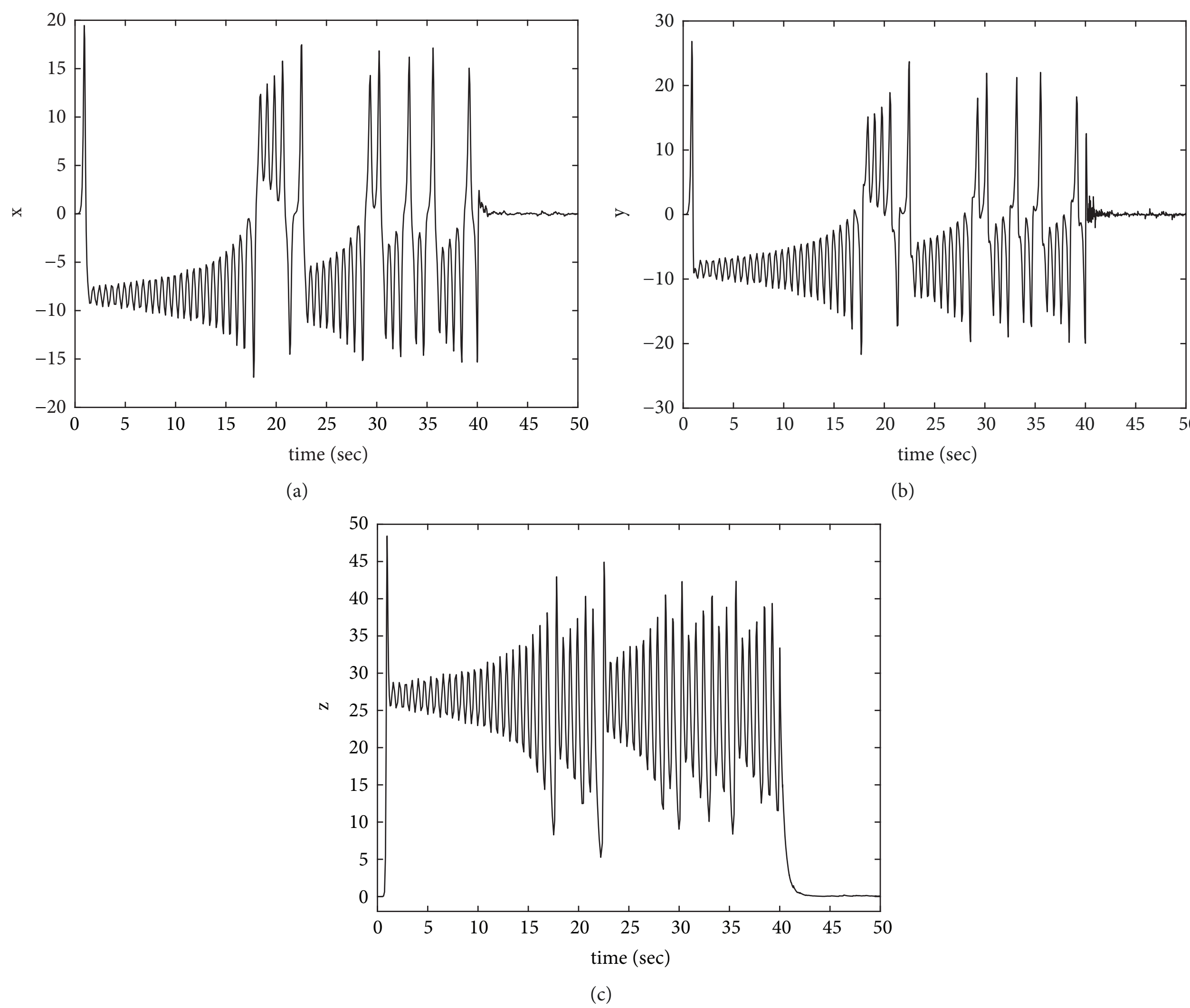

Figure 11: The time response of (a) x, (b) y, and (c) z state variables with the APPM activated at t = 40 seconds.

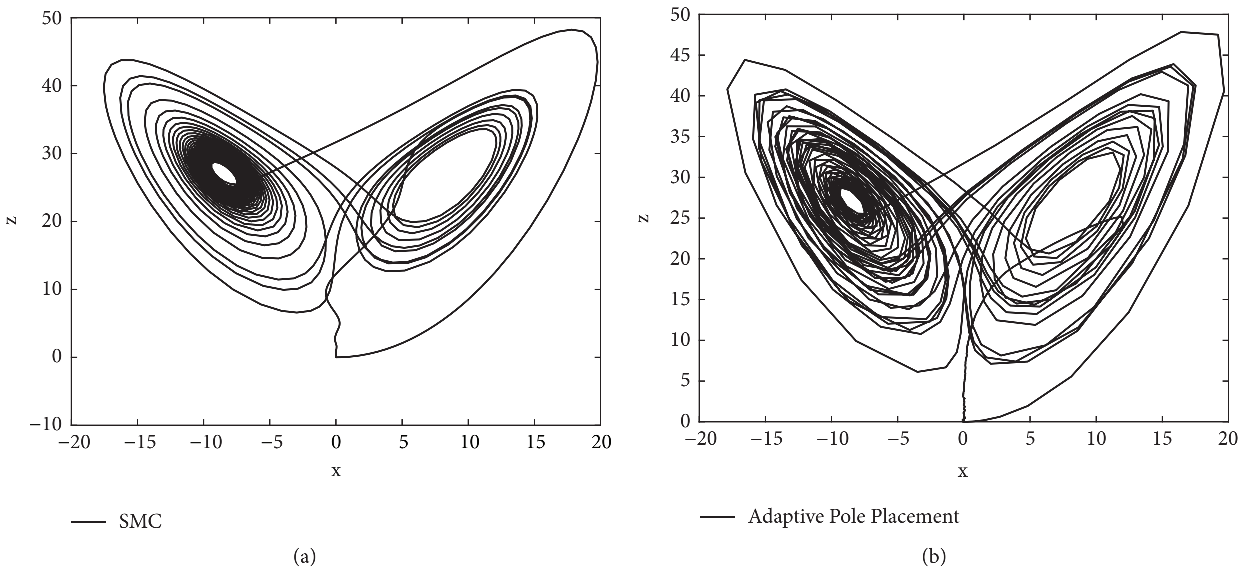

Figure 12: xz portraits obtained with SMC (a) and APP (b) control methods.

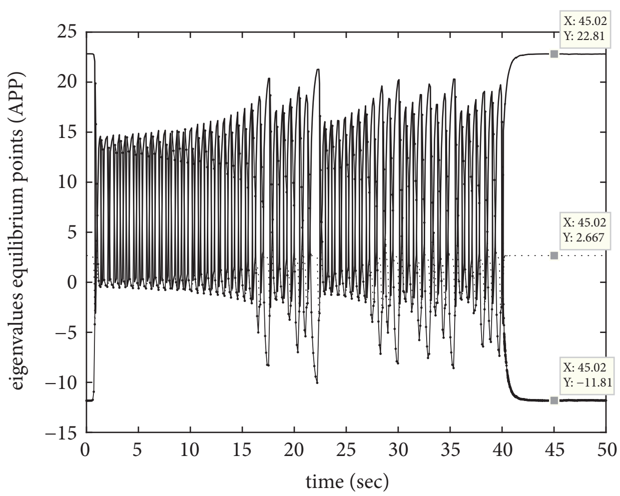

Figure 13: Eigenvalues values changes with controller active at t=40 seconds.

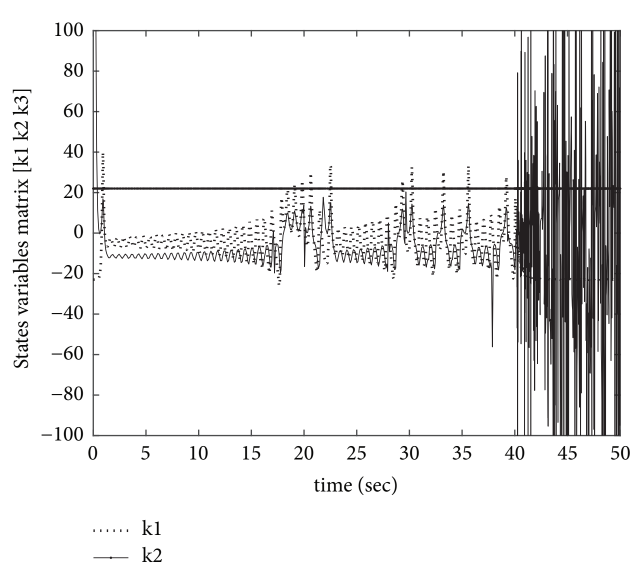

Figure 14: The change of adaptive control gain matrix $\mathbf{K}\,[\mathbf{k}_1 \quad \mathbf{k}_2 \quad \mathbf{k}_3]$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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